

Modelando objetos autogravitantes

```
> restart:with(plots):
```

Definimos las constantes para adimensionalizar la ecuación

$r = R * r1$  con  $R$  el radio exterior de la configuración y  $r1$  una variable adimensional

$P = P0 * P1$  con  $P0$  la presión central de la configuración y  $P1$  la función presión adimensional

$\rho = \rho0 * \rho1$  con  $\rho0$  la densidad central de la configuración y  $\rho1$  la función densidad adimensional

$m = M * m1$  con  $M$  la masa total de la configuración y  $m1$  la función masa adimensional

Las ecuaciones de estructura estelar para el caso newtoniano isótropo ( [http://eagle.phys.utk.edu/guidry/astro615/lectures/lecture\\_ch4.pdf](http://eagle.phys.utk.edu/guidry/astro615/lectures/lecture_ch4.pdf) ) quedan como

```
> Ecuac1 := diff(P1(r1), r1) + GG*Delta1*m1(r1)*rho1/r1^2=0;
```

$$Ecuac1 := \frac{d}{dr1} P1(r1) + \frac{GG \Delta1 m1(r1) \rho1}{r1^2} = 0 \quad (1)$$

```
> Ecuac2 := diff(m1(r1), r1) = Delta2*4*Pi*r1^2*rho1;
```

$$Ecuac2 := \frac{d}{dr1} m1(r1) = 4 \Delta2 \pi r1^2 \rho1 \quad (2)$$

Estas ecuaciones de estructura son adimensionales y pueden integrarse numéricamente con

$\Delta1 := \rho0 * Mtot / (R * P0)$

$\Delta2 := \rho0 * R^3 / Mtot$ ;

que utilizaremos más adelante

Ahora supongamos para este caso la ecuación politropa

```
> EcEstado := P1(r1) = K*rho1^gamma1;
```

$$EcEstado := P1(r1) = K \rho1^{\gamma1} \quad (3)$$

Adimensionalizamos y despejamos la densidad

```
> solrho := solve(EcEstado, rho1);
```

$$solrho := e^{\frac{\ln\left(\frac{P1(r1)}{K}\right)}{\gamma1}} \quad (4)$$

```
> rho1 := solrho;
```

$$\rho1 := e^{\frac{\ln\left(\frac{P1(r1)}{K}\right)}{\gamma1}} \quad (5)$$

Los parámetros de la ecuación politrópa

```
> K := 1.5; gamma1 := 4/3;
```

$$K := 1.5$$

$$\gamma1 := \frac{4}{3} \quad (6)$$

Incorporemos las constantes que adimensionalizan las ecuaciones

```
> Delta1 := rho0*M/(R*P0);
```

$$\Delta1 := \frac{\rho0 M}{R P0} \quad (7)$$

```
> Delta2 := rho0*R^3/M;
```

$$\Delta_2 := \frac{\rho_0 R^3}{M} \quad (8)$$

y consideremos los parámetros solares en el sistema MKS

```
> RadioSol := 6.955*10^8; MasaSol := 1.98855*10^30; DensidadCentralSol := 1.622*10^5; PresionCentralSol := 2.5*10^16;
```

$$RadioSol := 6.955000000 \cdot 10^8$$

$$MasaSol := 1.988550000 \cdot 10^{30}$$

$$DensidadCentralSol := 1.622000000 \cdot 10^5$$

$$PresionCentralSol := 2.500000000 \cdot 10^{16} \quad (9)$$

y la constante gravitacional MKS

```
> GG := 6.674*10^(-11); cc := 299792458;
```

$$GG := 6.674000000 \cdot 10^{-11}$$

$$cc := 299792458 \quad (10)$$

Las constantes para el Sol que se usaron para adimensionalizar

```
> M:=MasaSol;
R:=RadioSol;
P0:=PresionCentralSol;
rho0:=DensidadCentralSol;
```

$$M := 1.988550000 \cdot 10^{30}$$

$$R := 6.955000000 \cdot 10^8$$

$$P0 := 2.500000000 \cdot 10^{16}$$

$$\rho_0 := 1.622000000 \cdot 10^5 \quad (11)$$

Las ecuaciones de estructura para el caso newtoniana isótropo serán

```
> simplify(Ecuac1);
```

$$\frac{\left( \frac{d}{dr} P(r) \right) r^2 + 0.9134146656 m(r) P(r)^{3/4}}{r^2} = 0 \quad (12)$$

```
> simplify(Ecuac2);
```

$$\frac{d}{dr} m(r) = 254.4176034 r^2 P(r)^{3/4} \quad (13)$$

y los parámetros de ejecución

```
> r0 := 10^(-4); ifinal := 100; infinito := 10^(20):
```

$$r_0 := \frac{1}{10000}$$

$$ifinal := 100 \quad (14)$$

Se plantea la integración del sistema de ecuaciones de estructura y se varía la condición inicial para la presión central.

```
> for i from 0 to ifinal do
# se define el sistema de ecuaciones
SistemaPm := {Ecuac1, Ecuac2, P1(r0)= 5*i+1, m1(r0)=0.}:
funcionesPm := {P1(r1), m1(r1)}:
#Se integra el sistema de ecuaciones para condiciones iniciales
```

**variables en la presión**

```
solucion:= dsolve(SistemaPm,funcionesPm,numeric,events=[[P1(r1)=10^(-14),halt]],output=listprocedure):
```

```
#solucion:= dsolve(SistemaPm,funcionesPm,numeric,output=listprocedure):
```

```
# se almacena los valores resultados de la integración
```

```
radio := rhs(solucion[1]):presion := rhs(solucion[2]):masa := rhs(solucion[3]):
```

```
# se captura el último valor de la masa, generando una queja del sistema que desatendemos
```

```
masaTotal[i]:=masa(infinito):RadioExt[i]:=radio(infinito): Pnula[i]:=presion(infinito):
```

```
# a partir del valor de la presión central se calcula el densidad central
```

```
rhoCentral[i]:= evalf(subs(P1(r1)=presion(r0),rho1));
```

```
end do:
```

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Grafico los valores para puntos rhoCentral, masa total

```
> puntosMasarho0 := {seq([rhoCentral[i], masaTotal[i], i=1..ifinal])};  
> pointplot(puntosMasarho0, labels=[ "rhoCentral/rhocentralSol",  
  "M/Msol"], labeldirections = ["horizontal", "vertical"]);
```

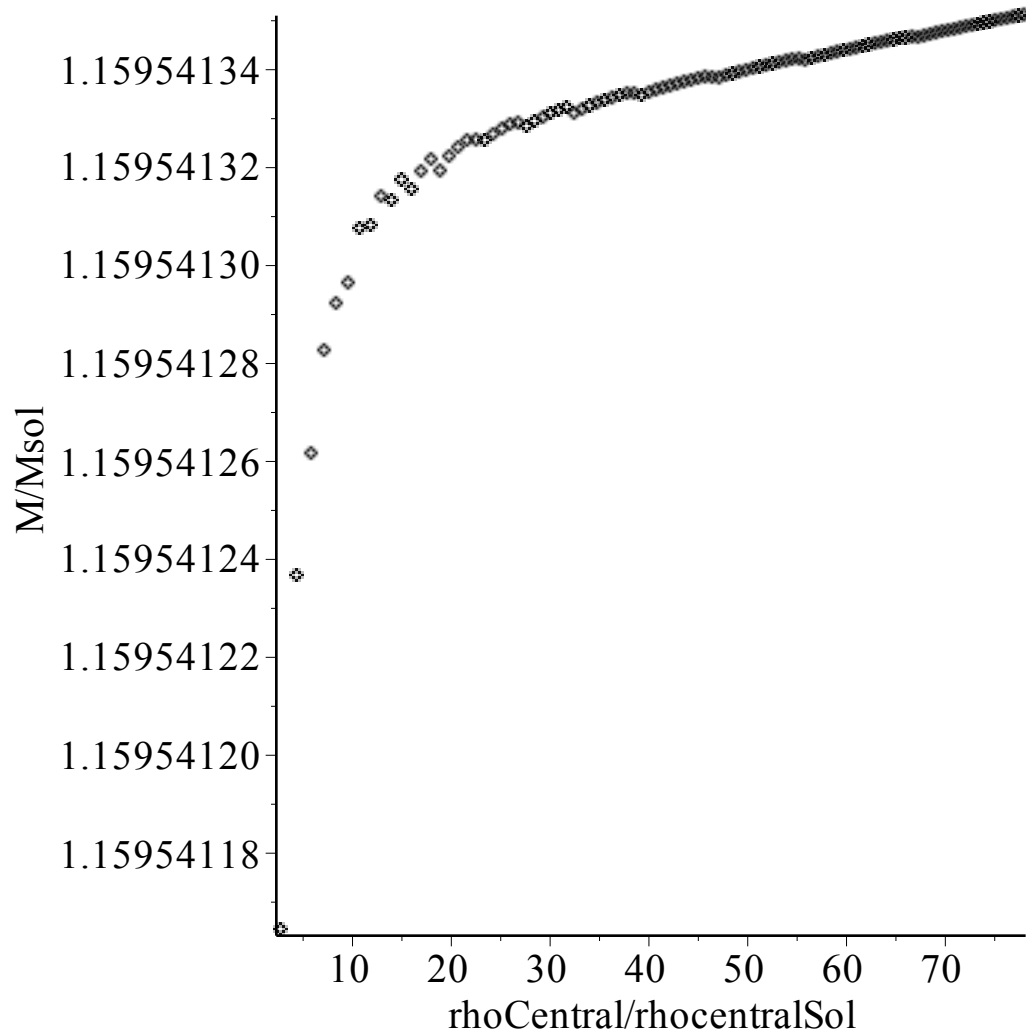


Grafico los puntos Radio exterior, masa total

```
> puntosMasaRadio:= {seq([RadioExt[i],masaTotal[i]],i=1..ifinal)}:
> pointplot(puntosMasaRadio,labels=[ "Radio/RadioSol", "M/Msol"],
  labeldirections = ["horizontal", "vertical"]);
```

