Dinámica y Movimiento Circular

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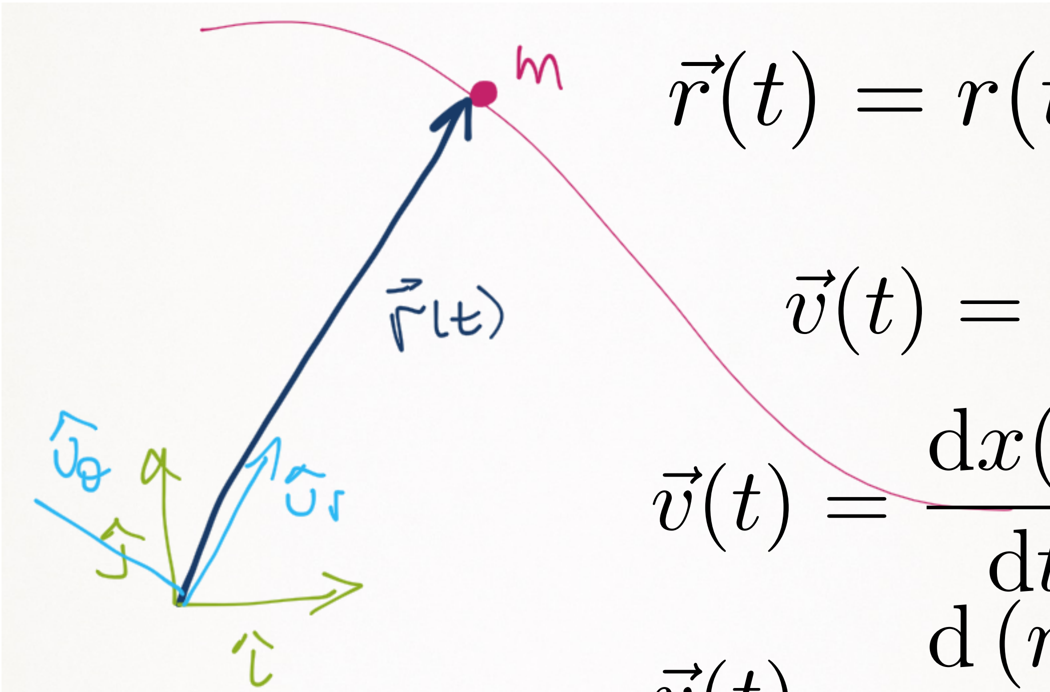
En general



$$\vec{r}(t) = r(t)\hat{\mathrm{u}}_r = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}$$

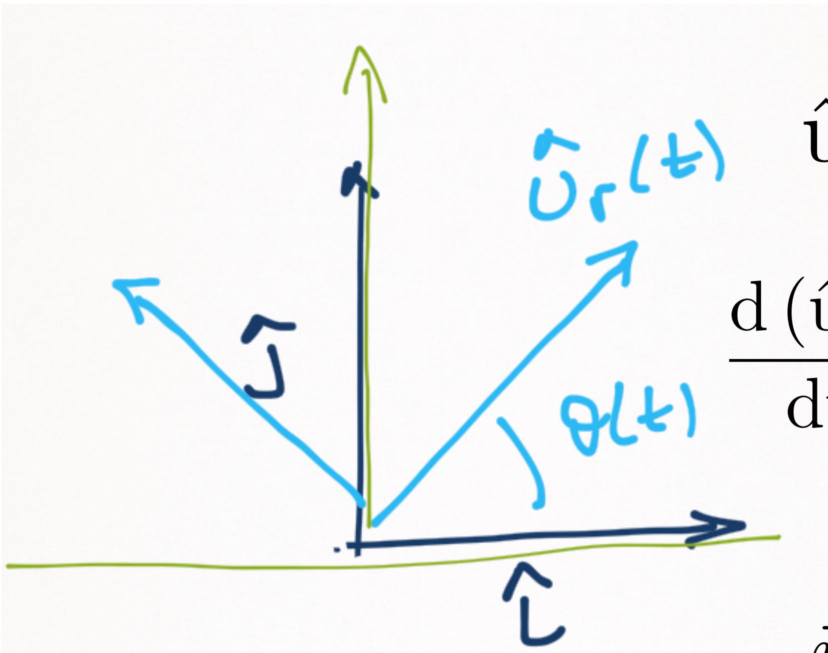
$$\vec{v}(t) = \frac{\mathrm{d}\vec{r}(t)}{\mathrm{d}t}$$

$$\vec{v}(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t}\hat{\mathrm{i}} + \frac{\mathrm{d}y(t)}{\mathrm{d}t}\hat{\mathrm{j}}$$

$$\vec{v}(t) = \frac{\mathrm{d}\left(r(t)\hat{\mathrm{u}}_r\right)}{\mathrm{d}t}$$

$$\vec{v}(t) == \frac{\mathrm{d}\left(r(t)\right)}{\mathrm{d}t}\hat{\mathrm{u}}_r + r(t)\frac{\mathrm{d}\left(\hat{\mathrm{u}}_r\right)}{\mathrm{d}t}$$

Componentes de vectores



$$\hat{\mathrm{u}}_r = \cos(\theta(t))\hat{\mathrm{i}} + \mathrm{sen}\,(\theta(t))\hat{\mathrm{j}}$$

$$\frac{\mathrm{d}\,(\hat{\mathrm{u}}_r)}{\mathrm{d}t} = \dot{\theta}(t)\underbrace{(-\mathrm{sen}(\theta(t))\hat{\mathrm{i}} + \cos(\theta(t))\hat{\mathrm{j}})}_{\hat{\mathrm{u}}_\theta}$$

$$\vec{v}(t) = \dot{r}(t)\hat{\mathrm{u}}_r + r(t)\dot{\theta}(t)\hat{\mathrm{u}}_\theta$$

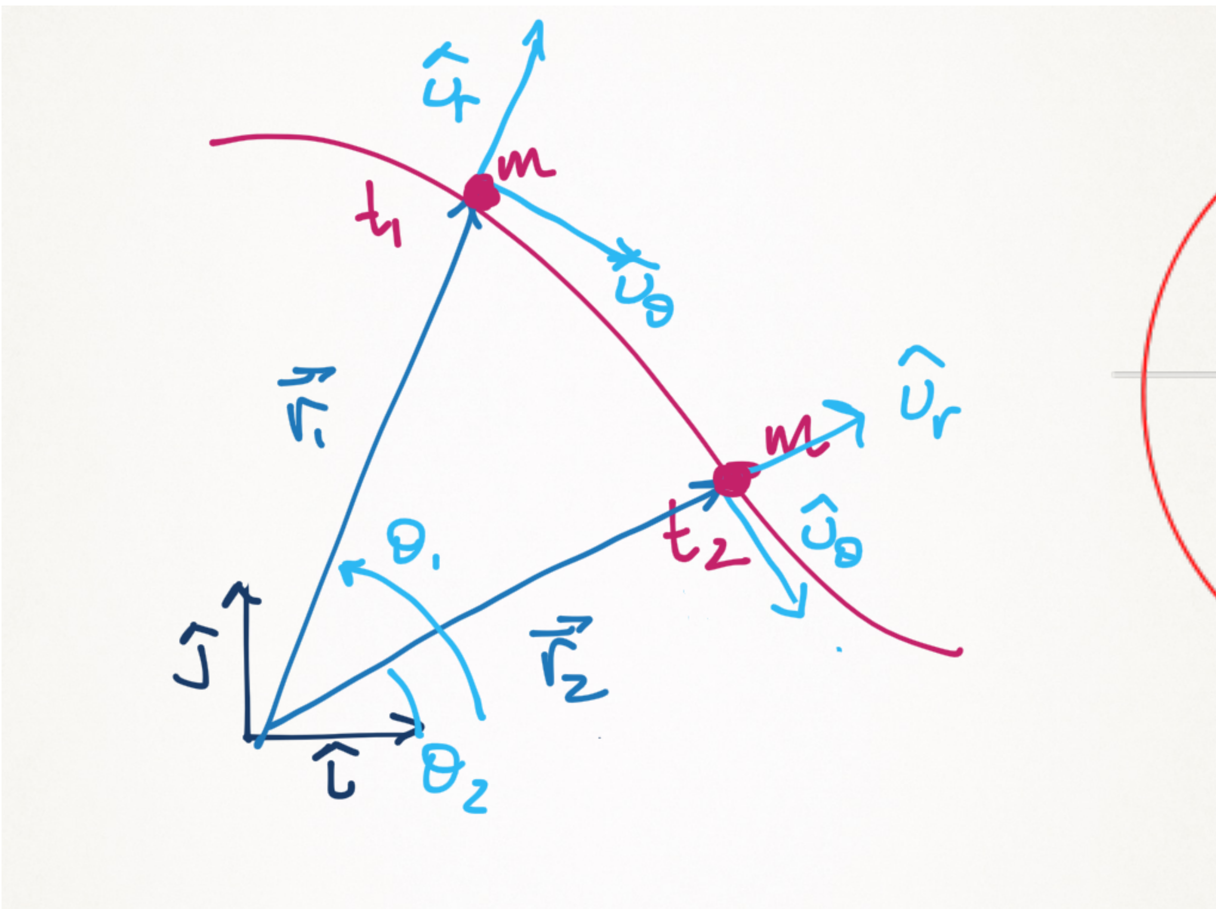
$$\vec{a}(t) = \ddot{r}(t)\hat{\mathrm{u}}_r + \dot{r}(t)\dot{\theta}(t)\hat{\mathrm{u}}_\theta + \dot{r}(t)\dot{\theta}(t)\hat{\mathrm{u}}_\theta + r(t)\ddot{\theta}(t)\hat{\mathrm{u}}_\theta + -r(t)\dot{\theta}^2(t)\hat{\mathrm{u}}_r$$

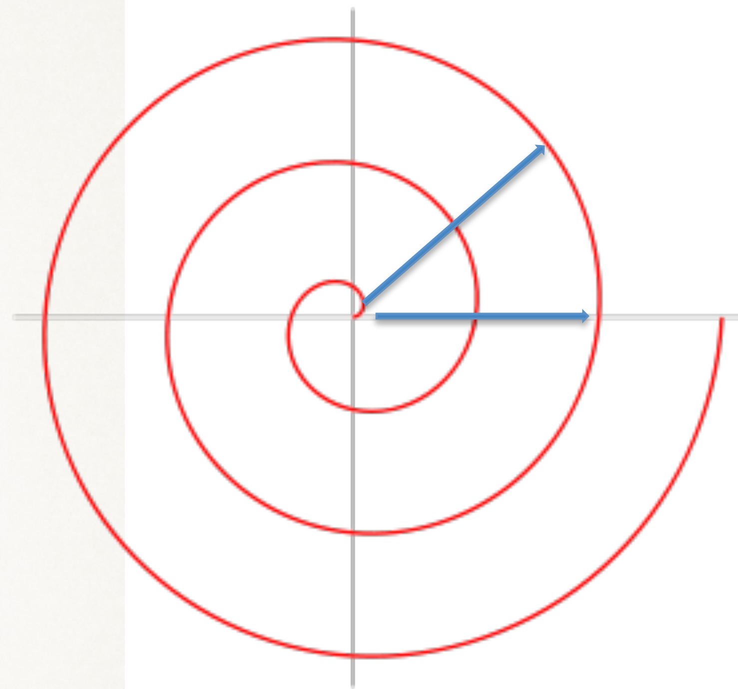
$$\vec{a}(t) = \left(\ddot{r}(t) - r(t)\dot{\theta}^2(t)\right)\hat{\mathrm{u}}_r + \left(2\dot{r}(t)\dot{\theta}(t) + r(t)\ddot{\theta}(t)\right)\hat{\mathrm{u}}_\theta$$

Aceleraciones radiales y tangenciales

$$\vec{a}(t) = \left(\ddot{r}(t) - r(t)\dot{\theta}^2(t)\right)\hat{\mathrm{u}}_r + \left(2\dot{r}(t)\dot{\theta}(t) + r(t)\ddot{\theta}(t)\right)\hat{\mathrm{u}}_\theta$$

Aceleración radial **Aceleración tangencial**



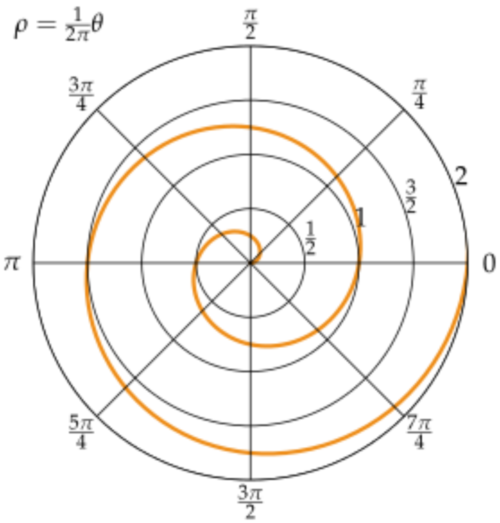


La espiral de Arquímides

$$\vec{a}(t) = \left(\ddot{r}(t) - r(t)\dot{\theta}^2(t)\right)\hat{u}_r + \left(2\dot{r}(t)\dot{\theta}(t) + r(t)\ddot{\theta}(t)\right)\hat{u}_\theta$$

$$r = b\theta \quad \text{con} \quad \omega = \frac{d\theta}{dt} = \dot{\theta} = cte \quad \text{y} \quad b = cte$$

$$r = b\omega t$$



$$\vec{a}(t) = \left(-r(t)\dot{\theta}^2(t)\right)\hat{u}_r + \left(2\dot{r}(t)\dot{\theta}(t)\right)\hat{u}_\theta$$

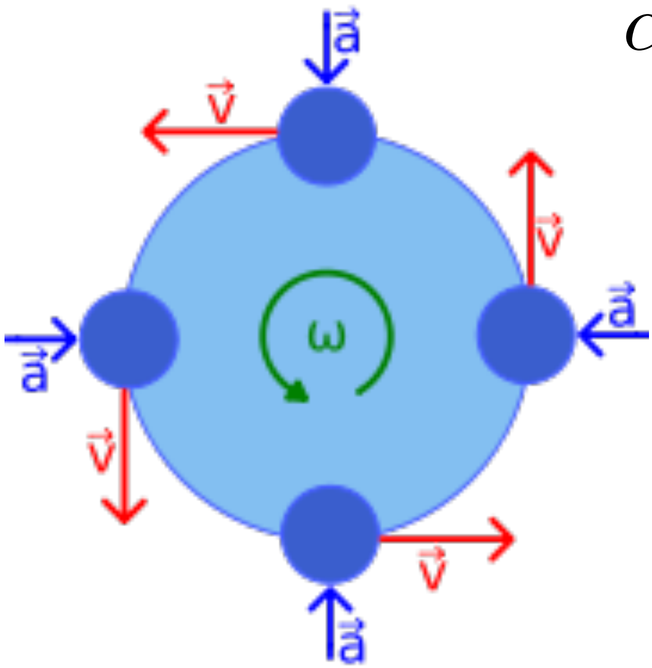
$$\vec{a}(t) = \left(-b\omega^3 t\right)\hat{u}_r + \left(2b\omega^2\right)\hat{u}_\theta = b\omega^2\left(-\omega t\hat{u}_r + 2\hat{u}_\theta\right)$$

$$\hat{u}_r = \cos(\theta)\hat{\imath} + \text{sen}(\theta)\hat{\jmath}$$

$$\hat{u}_\theta = -\text{sen}(\theta)\hat{\imath} + \cos(\theta)\hat{\jmath}$$

Movimiento circular

$$r = R = cte \quad \Rightarrow \vec{a}(t) = \left(-R\dot{\theta}(t)\right)\hat{\mathrm{u}}_r + \left(R\ddot{\theta}(t)\right)\hat{\mathrm{u}}_\theta$$



$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = cte$$

$$\omega = \omega_0 + \alpha t$$

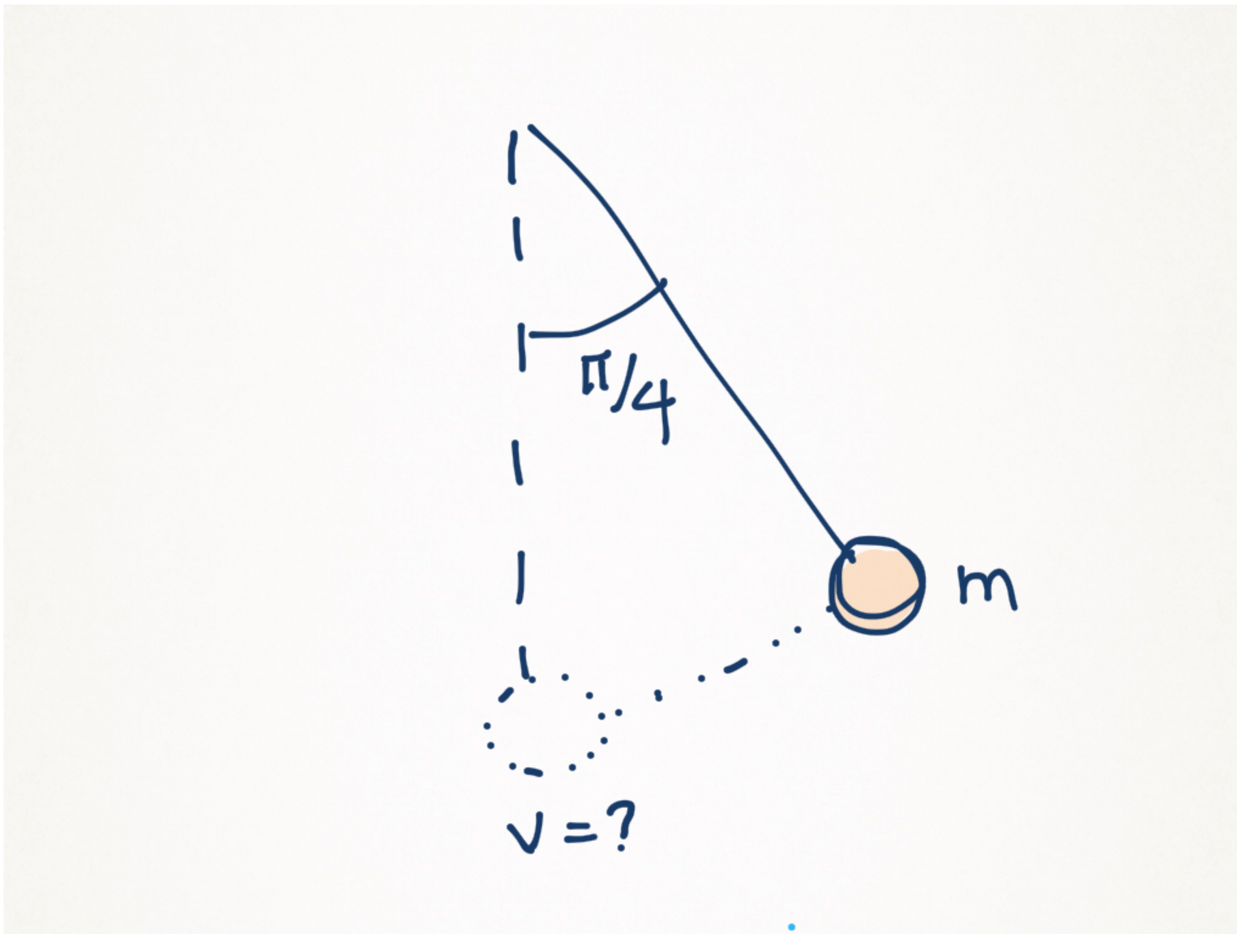
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$$

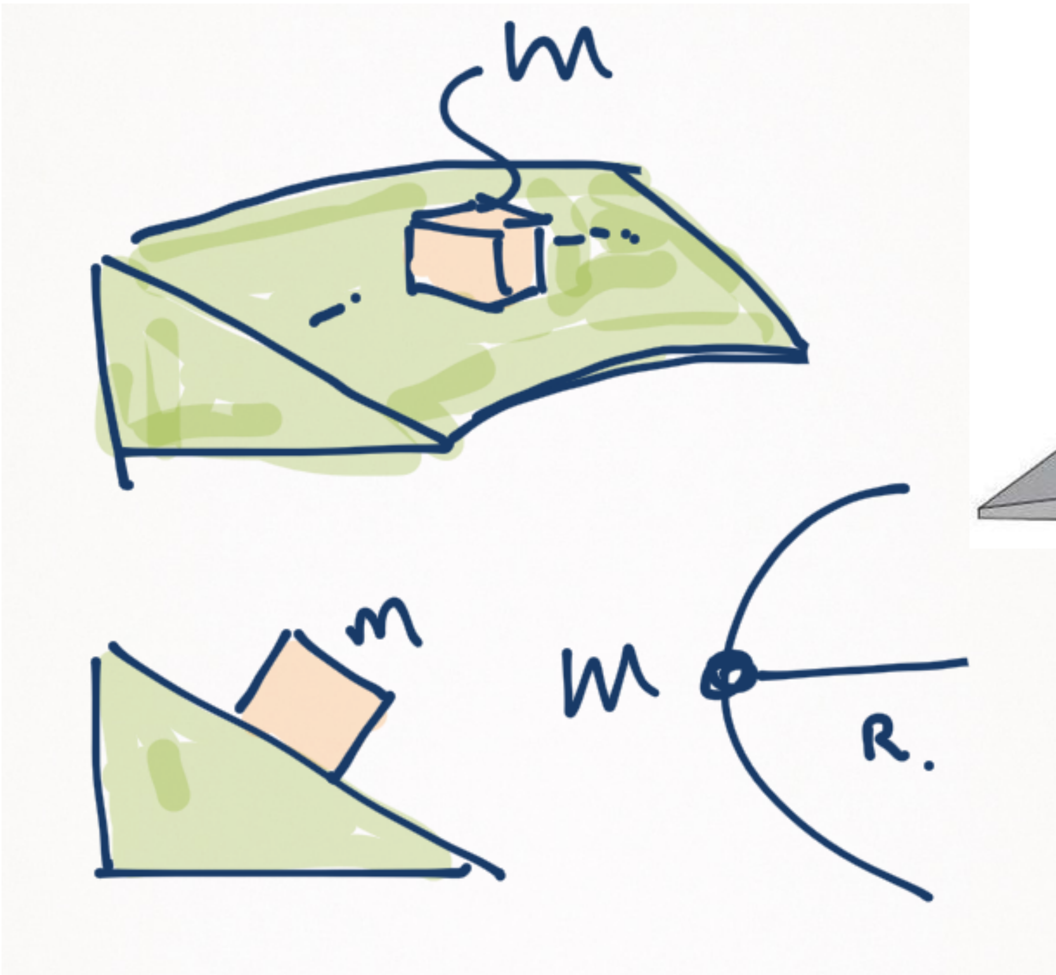
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

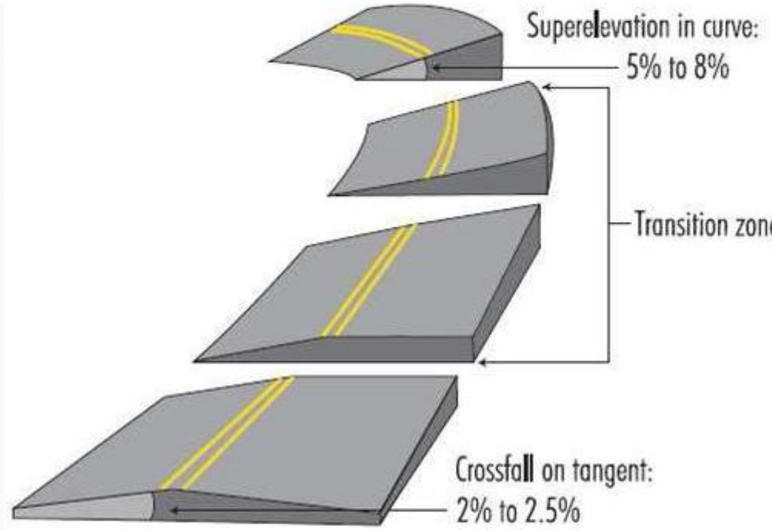
$$\theta = \theta_0 + \omega t - \frac{1}{2}\alpha t^2$$

Ejemplo 1: el péndulo



Ejemplo 2: el peralte





Vector velocidad angular

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{\omega} \times \vec{r} \equiv \frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{u}_\omega \times \vec{r}$$

$$\begin{aligned} \vec{r} &= x\hat{\imath} + y\hat{\jmath} + z\hat{k} \\ \vec{v} &= v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} \\ \vec{\omega} &= \omega_x\hat{\imath} + \omega_y\hat{\jmath} + \omega_z\hat{k} \end{aligned} \quad \Rightarrow \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}$$

$$\vec{v} = \hat{\imath}\left(\omega_y z - y\omega_z\right) + \hat{\jmath}\left(\omega_z x - z\omega_x\right) + \hat{k}\left(\omega_x z - x\omega_z\right)$$

$$\vec{v} = \hat{e}_1\left(\omega_2 r_3 - r_2\omega_3\right) + \hat{e}_2\left(\omega_3 r_1 - r_3\omega_1\right) + \hat{e}_3\left(\omega_1 r_2 - r_1\omega_2\right)$$

Velocidad angular y derivadas de vectores

$$\hat{\mathrm{u}}_r = \cos(\theta(t))\hat{\imath} + \mathrm{sen}\,(\theta(t))\hat{\jmath}$$

$$\frac{\mathrm{d}\,(\hat{\mathrm{u}}_r)}{\mathrm{d}t} = \dot{\theta}(t)\underbrace{(-\mathrm{sen}(\theta(t))\hat{\imath} + \cos(\theta(t))\hat{\jmath})}_{\hat{\mathrm{u}}_\theta} = \vec{\omega}\times\hat{\mathrm{u}}_r \quad \text{con} \quad \vec{\omega} = \dot{\theta}(t)\hat{k} \equiv \frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{k}$$

$$\frac{\mathrm{d}\,(\hat{\mathrm{u}}_r)}{\mathrm{d}t} = \hat{e}_1\,(-r_2\omega_3) + \hat{e}_2\,(\omega_3 r_1) = \hat{\imath}\left(-\mathrm{sen}(\theta)\dot{\theta}\right) + \hat{\jmath}\left(\dot{\theta}\cos(\theta)\right)$$

En general para 2D

$$\frac{\mathrm{d}\hat{\mathrm{e}}_1}{\mathrm{d}t} = \vec{\omega}\times\hat{\mathrm{e}}_1 \quad \text{y} \quad \frac{\mathrm{d}\hat{\mathrm{e}}_2}{\mathrm{d}t} = \vec{\omega}\times\hat{\mathrm{e}}_2 \qquad \text{con} \quad \vec{\omega} = \dot{\theta}\hat{\mathrm{e}}_3$$

$$\frac{\mathrm{d}\vec{c}}{\mathrm{d}t} = \frac{\mathrm{d}c_1}{\mathrm{d}t}\hat{\mathrm{e}}_1 + \frac{\mathrm{d}c_2}{\mathrm{d}t}\hat{\mathrm{e}}_2 + \frac{\mathrm{d}c_3}{\mathrm{d}t}\hat{\mathrm{e}}_3 + c_1\frac{\mathrm{d}\hat{\mathrm{e}}_1}{\mathrm{d}t} + c_2\frac{\mathrm{d}\hat{\mathrm{e}}_2}{\mathrm{d}t} + c_3\frac{\mathrm{d}\hat{\mathrm{e}}_3}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\vec{c}}{\mathrm{d}t} = \frac{\delta\vec{c}}{\delta t} + \vec{\omega}\times\vec{c} \quad \text{con} \quad \frac{\delta\vec{c}}{\delta t} = \frac{\mathrm{d}c_1}{\mathrm{d}t}\hat{\mathrm{e}}_1 + \frac{\mathrm{d}c_2}{\mathrm{d}t}\hat{\mathrm{e}}_2 + \frac{\mathrm{d}c_3}{\mathrm{d}t}\hat{\mathrm{e}}_3$$

Velocidad angular y derivadas de vectores

$$\frac{\mathrm{d}(\vec{\cdot})}{\mathrm{d}t} = \frac{\delta(\vec{\cdot})}{\delta t} + \vec{\omega} \times (\vec{\cdot})$$

Operador derivada vectorial

$$\frac{\mathrm{d}^2(\vec{\cdot})}{\mathrm{d}t^2} = \frac{\delta \frac{\mathrm{d}(\vec{\cdot})}{\mathrm{d}t}}{\delta t} + \vec{\omega} \times \frac{\mathrm{d}(\vec{\cdot})}{\mathrm{d}t}$$

Operador segimda derivada vectorial

$$\frac{\mathrm{d}^2(\vec{\cdot})}{\mathrm{d}t^2} = \frac{\mathrm{d}\left(\frac{\delta(\vec{\cdot})}{\delta t} + \vec{\omega} \times (\vec{\cdot})\right)}{\mathrm{d}t} = \frac{\delta\left(\frac{\delta(\vec{\cdot})}{\delta t} + \vec{\omega} \times (\vec{\cdot})\right)}{\delta t} + \vec{\omega} \times \left(\frac{\delta(\vec{\cdot})}{\delta t} + \vec{\omega} \times (\vec{\cdot})\right)$$

$$\frac{\mathrm{d}^2(\vec{\cdot})}{\mathrm{d}t^2} = \frac{\delta^2(\vec{\cdot})}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times (\vec{\cdot}) + 2\vec{\omega} \times \frac{\delta(\vec{\cdot})}{\delta t} + \vec{\omega} \times \left(\vec{\omega} \times (\vec{\cdot})\right)$$

Velocidad angular y sistemas de referencia

$$\frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\delta\vec{\omega}}{\delta t} + \vec{\omega} \times \vec{\omega} \quad \Rightarrow \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\delta\vec{\omega}}{\delta t}$$

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\delta\vec{r}}{\delta t} + \vec{\omega} \times \vec{r}$$

Variación del módulo del vector posición

Variación por rotación del sistema de coordenadas

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\delta\vec{v}}{\delta t} + \vec{\omega} \times \vec{v}$$

$$\vec{a} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\delta^2\vec{r}}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + 2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$