Sistemas de referencia no inerciales

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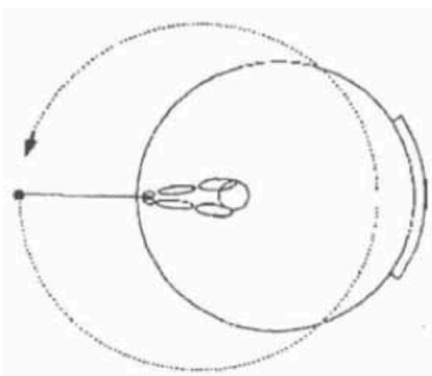
Inerciales antes de no inerciales

$$\vec{r}'(t) = \vec{R}_{oo'}(t) + \vec{r}(t)$$

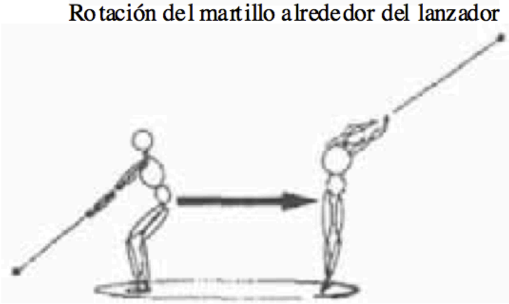
$$\vec{v}'(t) = \vec{V}_{oo'}(t) + \vec{v}(t)$$

Transformaciones de Galileo



Rotación del martillo alrededor del lanzador

Plano de movimiento del martillo



Movimiento de traslación

Movimiento total

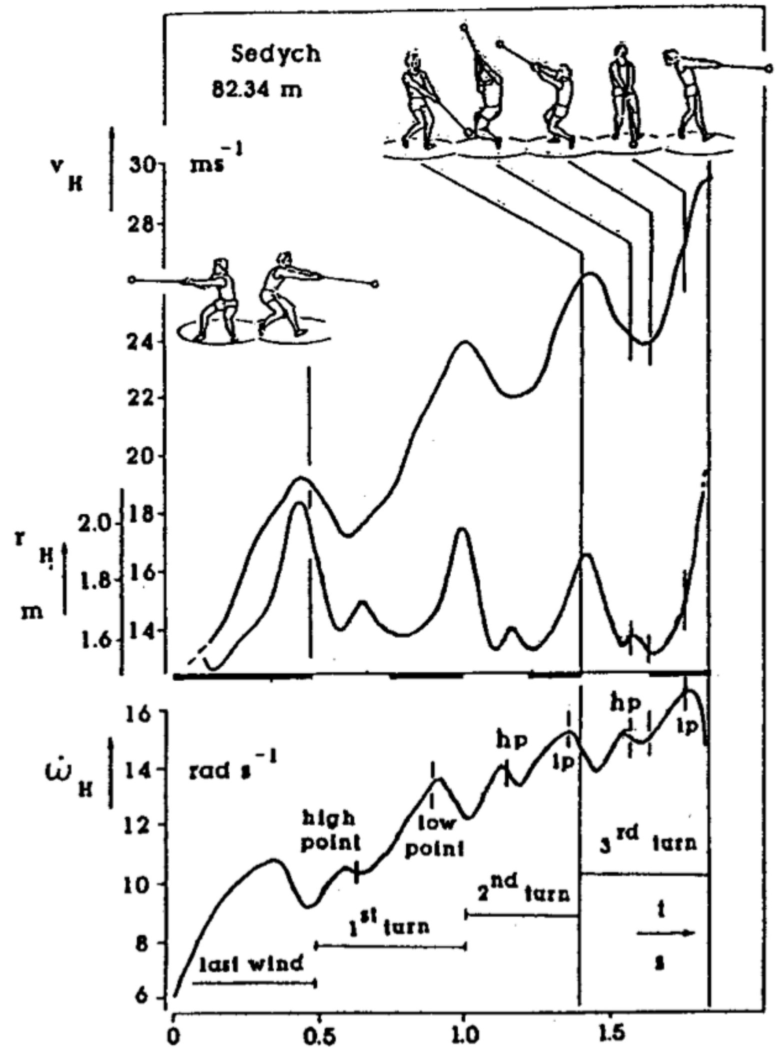
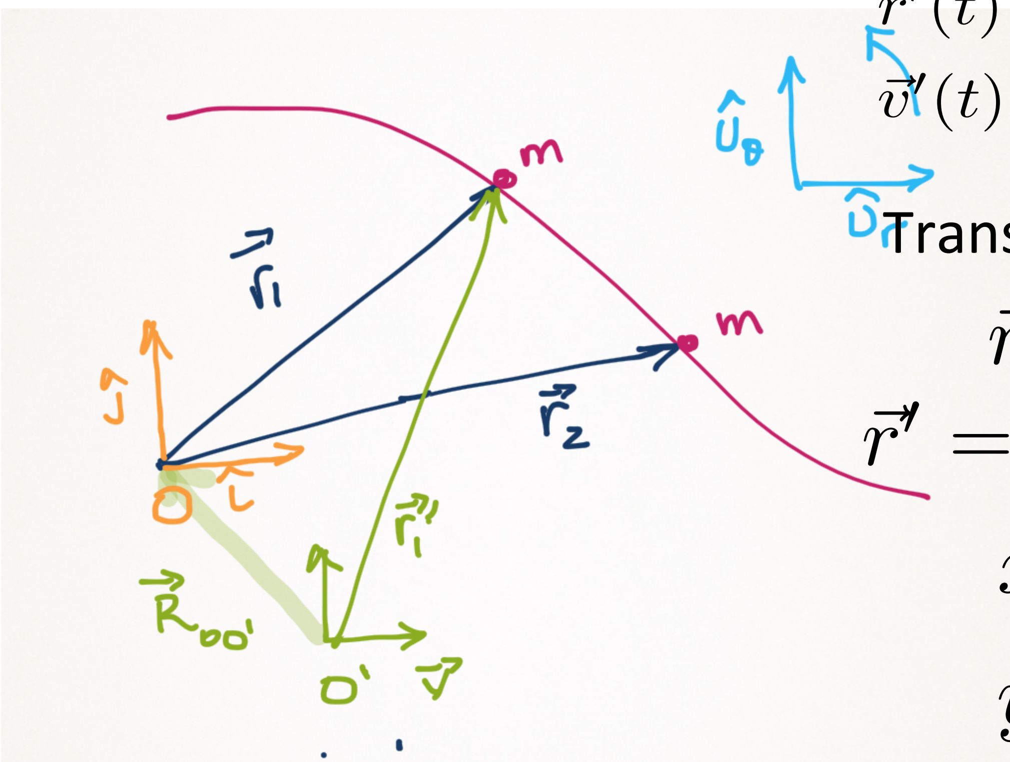


FIG. 2: Time related changes of the velocity, radius and angular velocity of the hammer.

1era aproximación: Rotación + Traslación 2D



$$\vec{r}'(t) = \vec{R}_{oo'}(t) + \vec{r}(t)$$

$$\vec{v}'(t) = \vec{V}_{oo'}(t) + \vec{v}(t)$$

Transformaciones de Galileo

$$\vec{r} = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

$$\vec{r}' = x'(t)\,\hat{\imath}' + y'(t)\,\hat{\jmath}'$$

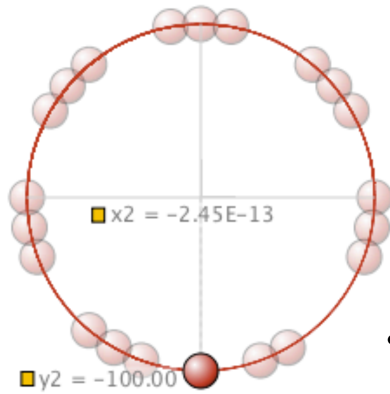
$$x(t) = R\cos(\omega t)$$

$$y(t) = R\,\mathrm{sen}(\omega t)$$

$$x'(t) = R\cos(\omega t) + V_{oo'}t$$

$$y'(t) = R\,\mathrm{sen}(\omega t)$$

1era aproximación: Rotación + Traslación 2D



$$x(t) = R\cos(\omega t)$$

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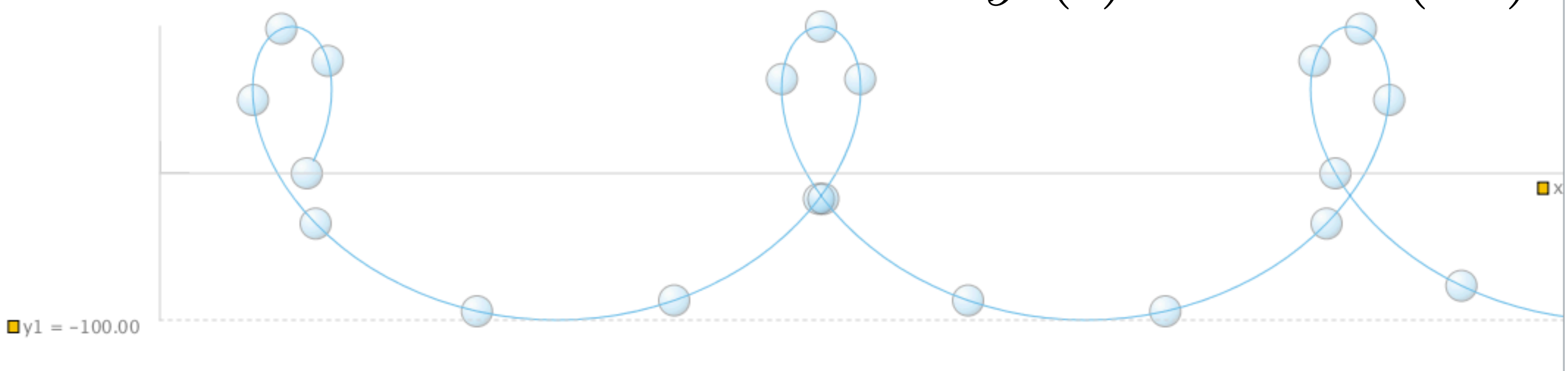
$$x'(t) = R\cos(\omega t) + V_{oo'}t$$

$$y'(t) = R\,\mathrm{sen}(\omega t)$$

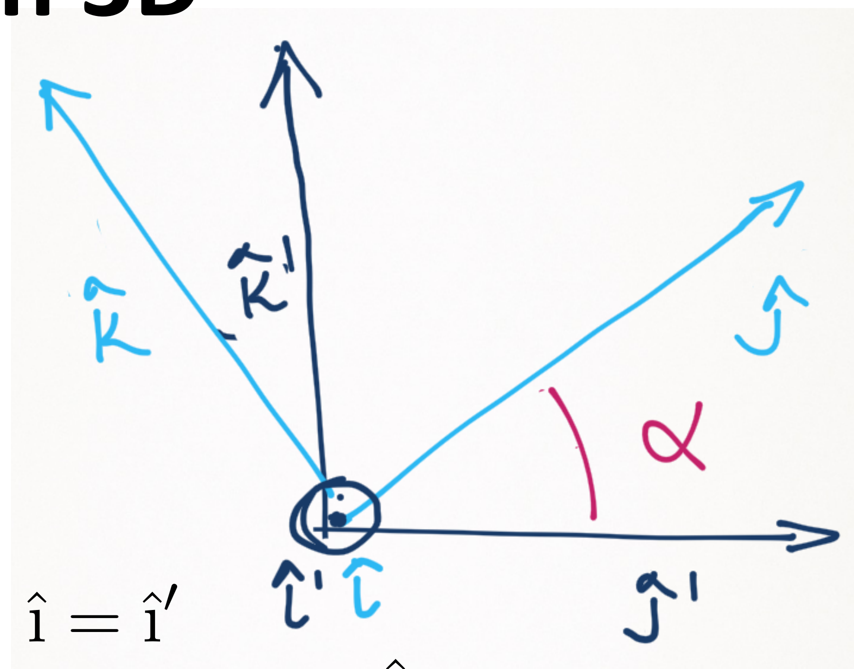


2da aproximación: Rotación + Traslación 3D

$$\vec{r} = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} + z(t)\,\hat{k}$$

$$\vec{r}' = x'(t)\,\hat{\imath}' + y'(t)\,\hat{\jmath}' + z'(t)\,\hat{k}'$$

$$\vec{r}'(t) = \vec{r}(t) + \vec{\mathcal{R}}_{oo'}(t)$$



$$\hat{\imath}' \Rightarrow x'(t) = \tilde{x}(t) + X_{oo'}(t)$$

$$\hat{\jmath}' \Rightarrow y'(t) = \tilde{y}(t) + Y_{oo'}(t)$$

$$\hat{k}' \Rightarrow z'(t) = \tilde{z}(t) + Z_{oo'}(t)$$

$$\hat{\imath} = \hat{\imath}'$$

$$\hat{\jmath} = \hat{\jmath}'\cos(\alpha) + \hat{k}'\,\mathrm{sen}(\alpha)$$

$$\hat{k} = -\hat{\jmath}'\,\mathrm{sen}(\alpha) + \hat{k}'\cos(\alpha)$$

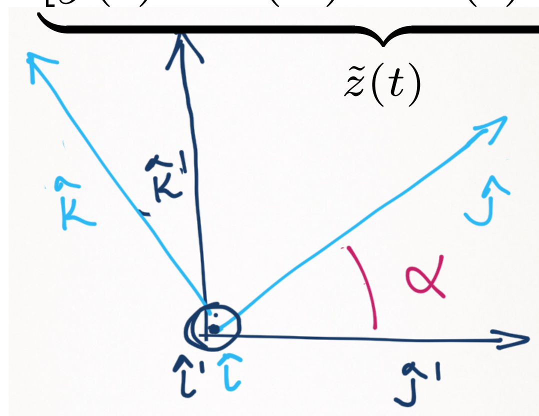
$$\vec{r} = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} + z(t)\hat{k} = \tilde{x}(t)\,\hat{\imath}' + \tilde{y}(t)\,\hat{\jmath}' + \tilde{z}(t)\hat{k}'$$

2da aproximación: Rotación + Traslación 3D

$$\vec{r} = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} + z(t)\hat{k} = \tilde{x}(t)\,\hat{\imath}' + \tilde{y}(t)\,\hat{\jmath}' + \tilde{z}(t)\hat{k}'$$

$$\vec{r} = x(t)\,\hat{\imath}' + y(t)\,\left(\hat{\jmath}'\cos(\alpha) + \hat{k}'\,\mathrm{sen}(\alpha)\right) + z(t)\left(-\hat{\jmath}'\,\mathrm{sen}(\alpha) + \hat{k}'\cos(\alpha)\right)$$

$$\vec{r} = \underbrace{x(t)}_{\tilde{x}(t)}\,\hat{\imath}' + \underbrace{[y(t)\cos(\alpha) - z(t)\,\mathrm{sen}(\alpha)]\,\hat{\jmath}'}_{\tilde{y}(t)} + \underbrace{[y(t)\,\mathrm{sen}(\alpha) + z(t)\cos(\alpha)]}_{\tilde{z}(t)}\,\hat{k}'$$



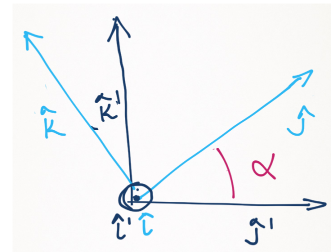
$$\vec{r}'(t) = \vec{r}(t) + \vec{\mathcal{R}}_{oo'}(t)$$

$$\hat{\imath}' \Rightarrow x'(t) = \tilde{x}(t) = x(t)$$

$$\hat{\jmath}' \Rightarrow y'(t) = \tilde{y}(t) + Y_{oo'}(t) = y(t)\cos(\alpha) - z(t)\,\mathrm{sen}(\alpha) + V_{oo'}t$$

$$\hat{k}' \Rightarrow z'(t) = \tilde{z}(t) + Z_{oo'}(t) = y(t)\,\mathrm{sen}(\alpha) + z(t)\cos(\alpha)$$

2da aproximación: Rotación + Traslación 3D



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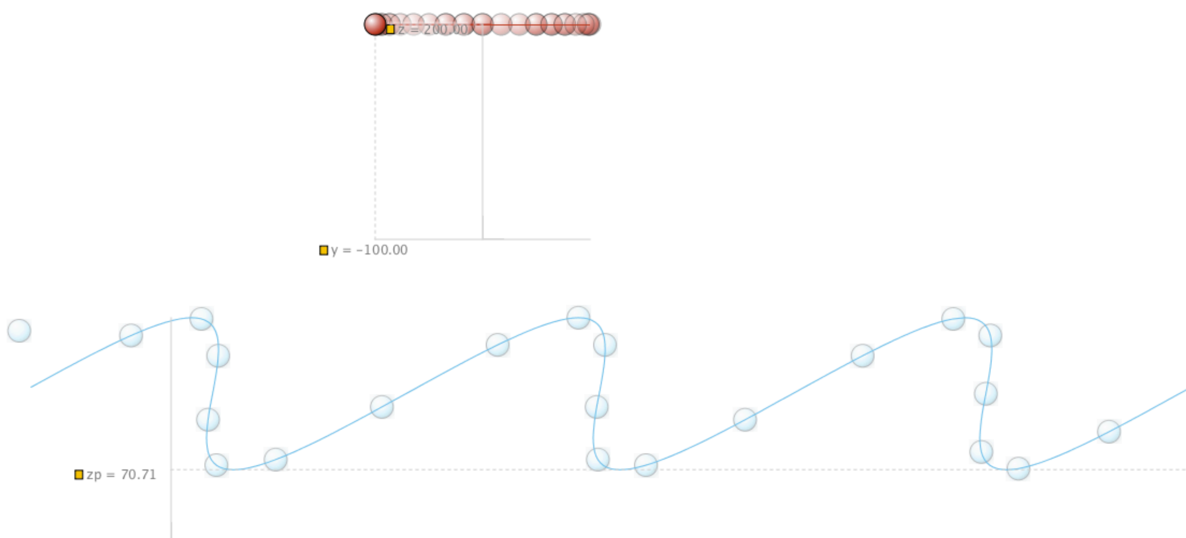
$$\hat{k}' \Rightarrow z'(t) = \tilde{z}(t) + Z_{oo'}(t) = y(t)\,\mathrm{sen}(\alpha) + z(t)\cos(\alpha)$$



Sistemas no inerciales

$$\vec{r}'(t) = \vec{\mathcal{R}}_{oo'}(t) + \vec{r}(t)$$

$$\vec{v}'(t) = \vec{\mathcal{V}}_{oo'}(t) + \vec{v}(t)$$

$$\vec{a}'(t) = \vec{\mathcal{A}}_{oo'}(t) + \vec{a}(t)$$