Fuerza Inerciales

Luis A. Núñez

Esc. Física

Universidad Industrial de Santander

Escuela
de Física

Universidad
industrial de
Santander

Grupo Halley
Astronomía y Ciencias Aeroespaciales

COMPLEJO
ASTRONÓMICO
Grupo Halley

Velocidad angular y derivadas de vectores

$$\hat{\mathrm{u}}_r = \cos(\theta(t))\hat{\imath} + \operatorname{sen}(\theta(t))\hat{\jmath}$$

$$\frac{\mathrm{d}\left(\hat{\mathrm{u}}_r\right)}{\mathrm{d}t} = \dot{\theta}(t)\underbrace{\left(-\operatorname{sen}(\theta(t))\hat{\imath} + \cos(\theta(t))\hat{\jmath}\right)}_{\hat{\mathrm{u}}_\theta} = \vec{\omega} \times \hat{\mathrm{u}}_r \quad \text{con} \quad \vec{\omega} = \dot{\theta}(t)\hat{k} \equiv \frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{k}$$

$$\frac{\mathrm{d}\left(\hat{\mathrm{u}}_r\right)}{\mathrm{d}t} = \hat{e}_1\left(-r_2\omega_3\right) + \hat{e}_2\left(\omega_3 r_1\right) = \hat{\imath}\left(-\operatorname{sen}(\theta)\dot{\theta}\right) + \hat{\jmath}\left(\dot{\theta}\cos(\theta)\right)$$

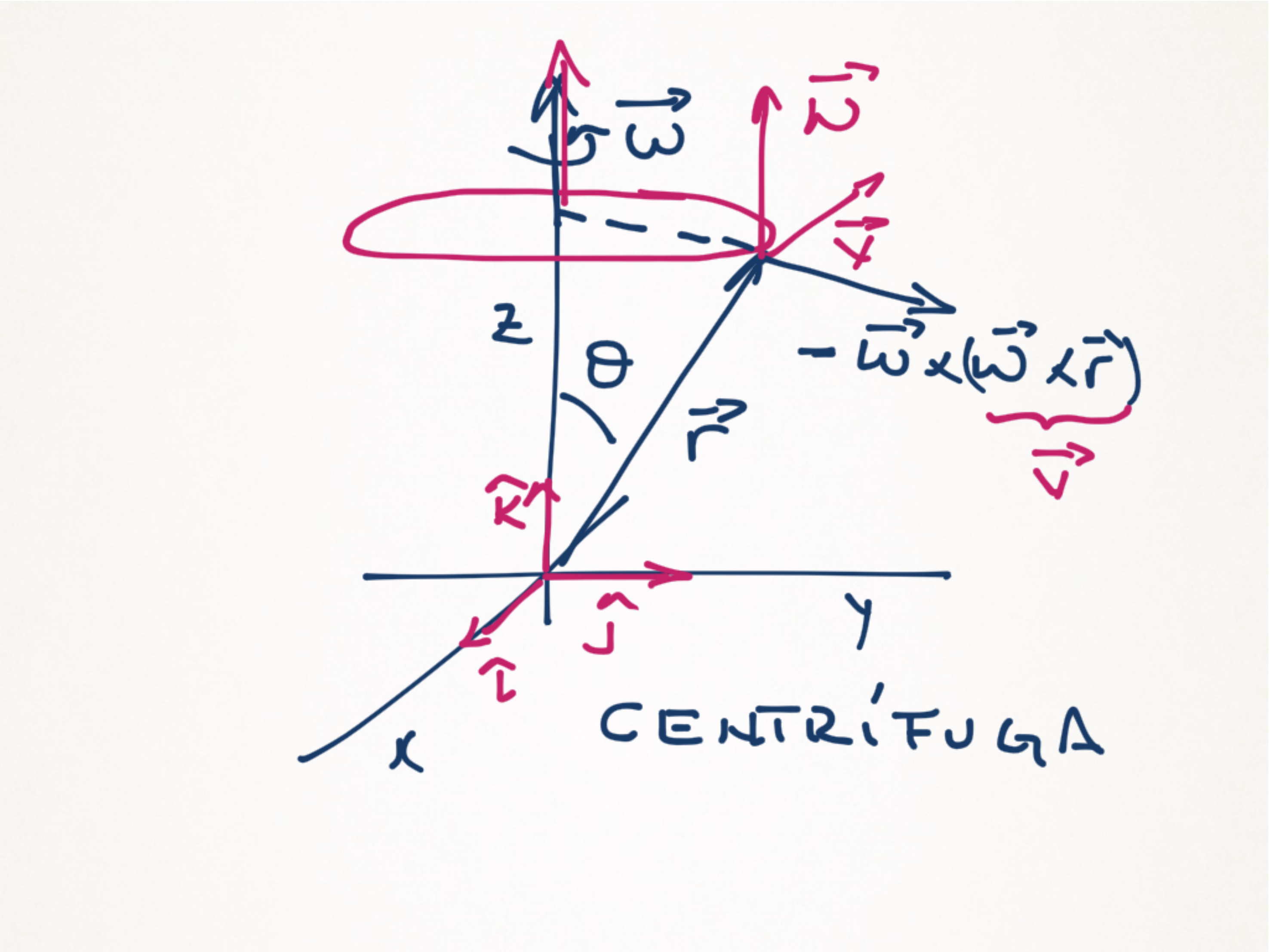
En general para 2D

$$\frac{\mathrm{d}\hat{\mathrm{e}}_1}{\mathrm{d}t} = \vec{\omega} \times \hat{\mathrm{e}}_1 \quad \text{y} \quad \frac{\mathrm{d}\hat{\mathrm{e}}_2}{\mathrm{d}t} = \vec{\omega} \times \hat{\mathrm{e}}_2 \qquad \text{con} \quad \vec{\omega} = \dot{\theta}\hat{\mathrm{e}}_3$$

$$\frac{\mathrm{d}\vec{c}}{\mathrm{d}t} = \frac{\mathrm{d}c_1}{\mathrm{d}t}\hat{\mathrm{e}}_1 + \frac{\mathrm{d}c_2}{\mathrm{d}t}\hat{\mathrm{e}}_2 + \frac{\mathrm{d}c_3}{\mathrm{d}t}\hat{\mathrm{e}}_3 + c_1\frac{\mathrm{d}\hat{\mathrm{e}}_1}{\mathrm{d}t} + c_2\frac{\mathrm{d}\hat{\mathrm{e}}_2}{\mathrm{d}t} + c_3\frac{\mathrm{d}\hat{\mathrm{e}}_3}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\vec{c}}{\mathrm{d}t} = \frac{\delta\vec{c}}{\delta t} + \vec{\omega} \times \vec{c} \quad \text{con} \quad \frac{\delta\vec{c}}{\delta t} = \frac{\mathrm{d}c_1}{\mathrm{d}t}\hat{\mathrm{e}}_1 + \frac{\mathrm{d}c_2}{\mathrm{d}t}\hat{\mathrm{e}}_2 + \frac{\mathrm{d}c_3}{\mathrm{d}t}\hat{\mathrm{e}}_3$$

¿Centrífuga?



$\vec{\omega}$
$\vec{\omega}$
$\vec{v}$
z
θ
$-\vec{\omega}\times(\vec{\omega}\times\vec{r})$
$\vec{r}$
$\vec{v}$
$\hat{k}$
$\hat{j}$
y
$\hat{i}$
x
CENTRÍFUGA

Velocidad angular y derivadas de vectores

En general

$$\frac{\mathrm{d}\vec{(\cdot)}}{\mathrm{d}t} = \frac{\delta\vec{(\cdot)}}{\delta t} + \vec{\omega} \times \vec{(\cdot)}$$

Operador derivada vectorial

$$\frac{\mathrm{d}^2\vec{(\cdot)}}{\mathrm{d}t^2} = \frac{\delta \frac{\mathrm{d}\vec{(\cdot)}}{\mathrm{d}t}}{\delta t} + \vec{\omega} \times \frac{\mathrm{d}\vec{(\cdot)}}{\mathrm{d}t}$$

Operador segimda derivada vectorial

$$\frac{\mathrm{d}^2\vec{(\cdot)}}{\mathrm{d}t^2} = \frac{\mathrm{d}\left(\frac{\delta\vec{(\cdot)}}{\delta t} + \vec{\omega} \times \vec{(\cdot)}\right)}{\mathrm{d}t} = \frac{\delta\left(\frac{\delta\vec{(\cdot)}}{\delta t} + \vec{\omega} \times \vec{(\cdot)}\right)}{\delta t} + \vec{\omega} \times \left(\frac{\delta\vec{(\cdot)}}{\delta t} + \vec{\omega} \times \vec{(\cdot)}\right)$$

$$\frac{\mathrm{d}^2\vec{(\cdot)}}{\mathrm{d}t^2} = \frac{\delta^2\vec{(\cdot)}}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times \vec{(\cdot)} + 2\vec{\omega} \times \frac{\delta\vec{(\cdot)}}{\delta t} + \vec{\omega} \times \left(\vec{\omega} \times \vec{(\cdot)}\right)$$

Variación del módulo
del vector posición respecto al
Sistema móvil

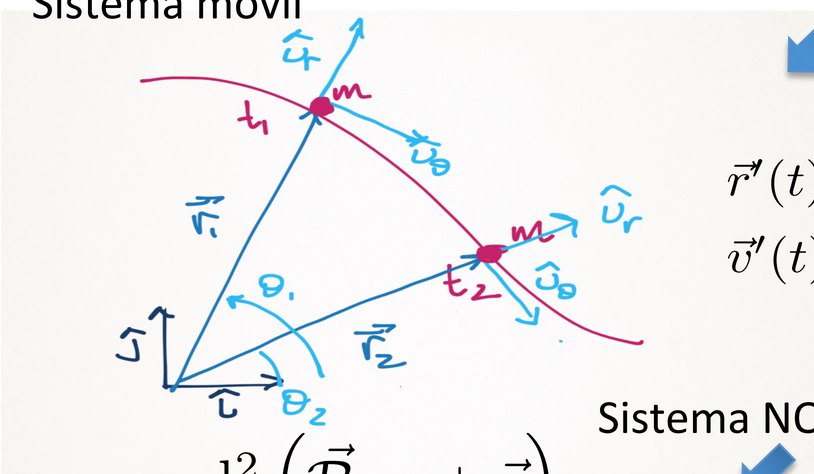
Variación por rotación
del sistema de coordenadas móviles

La velocidad angular y la aceleración lineal

$$\vec{a} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\delta^2\vec{r}}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + 2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Variación del módulo
del vector posición respecto al
Sistema móvil

Variación por rotación
del sistema de coordenadas móviles



$$\vec{r}'(t) = \vec{R}_{oo'}(t) + \vec{r}(t)$$
$$\vec{v}'(t) = \vec{V}_{oo'}(t) + \vec{v}(t)$$

Sistema NO Inercial

$$\frac{\mathrm{d}^2\vec{r}_I}{\mathrm{d}t^2} = \frac{\mathrm{d}^2\left(\vec{\mathcal{R}}_{oo'} + \vec{r}\right)}{\mathrm{d}t^2} = \frac{\delta^2\vec{r}}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + 2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \frac{\mathrm{d}^2\vec{\mathcal{R}}_{oo'}}{\mathrm{d}t^2}$$

Sistema Inercial

¿Fuerzas ficticias? ¿Fuerzas no inerciales?

$$m\frac{\mathrm{d}^2\vec{r}_I}{\mathrm{d}t^2} = m\frac{\delta^2\vec{r}}{\delta t^2} + m\frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + m2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + m\vec{\omega} \times (\vec{\omega} \times \vec{r}) + \frac{\mathrm{d}^2\vec{\mathcal{R}}_{oo'}}{\mathrm{d}t^2}$$

Supongamos que el sistema está aislado respecto a un sistema INERCIAL de coordenadas

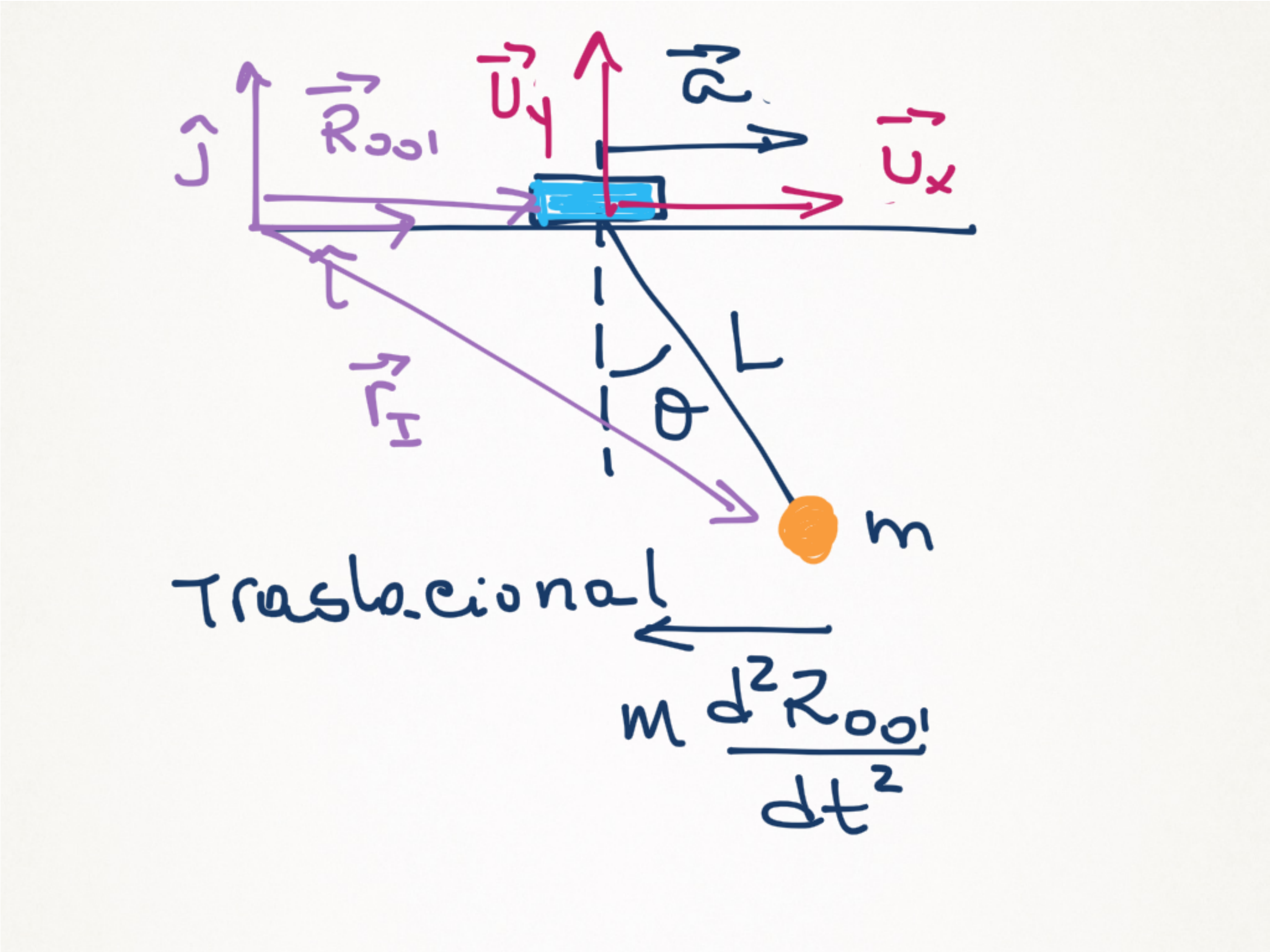
$$0 = m\frac{\delta^2\vec{r}}{\delta t^2} + m\frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + m2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + m\vec{\omega} \times (\vec{\omega} \times \vec{r}) + \frac{\mathrm{d}^2\vec{\mathcal{R}}_{oo'}}{\mathrm{d}t^2}$$

Las fuerzas que mide un sistema NO INERCIAL serán:

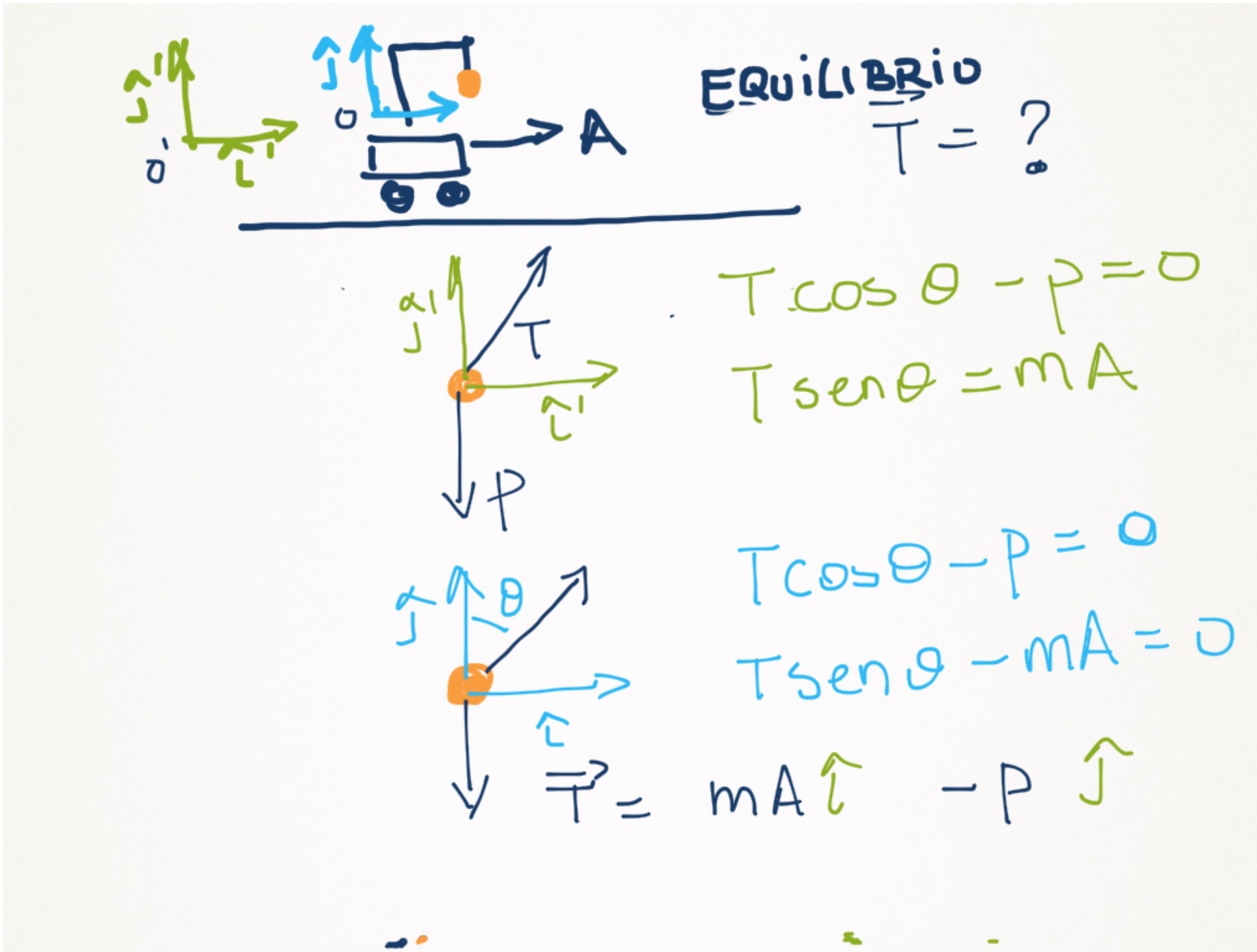
$$m\frac{\delta^2\vec{r}}{\delta t^2} = -m\frac{\delta\vec{\omega}}{\delta t} \times \vec{r} - m2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - \frac{\mathrm{d}^2\vec{\mathcal{R}}_{oo'}}{\mathrm{d}t^2}$$

Azimutal | Coriolis | Centrífuga | Traslacional

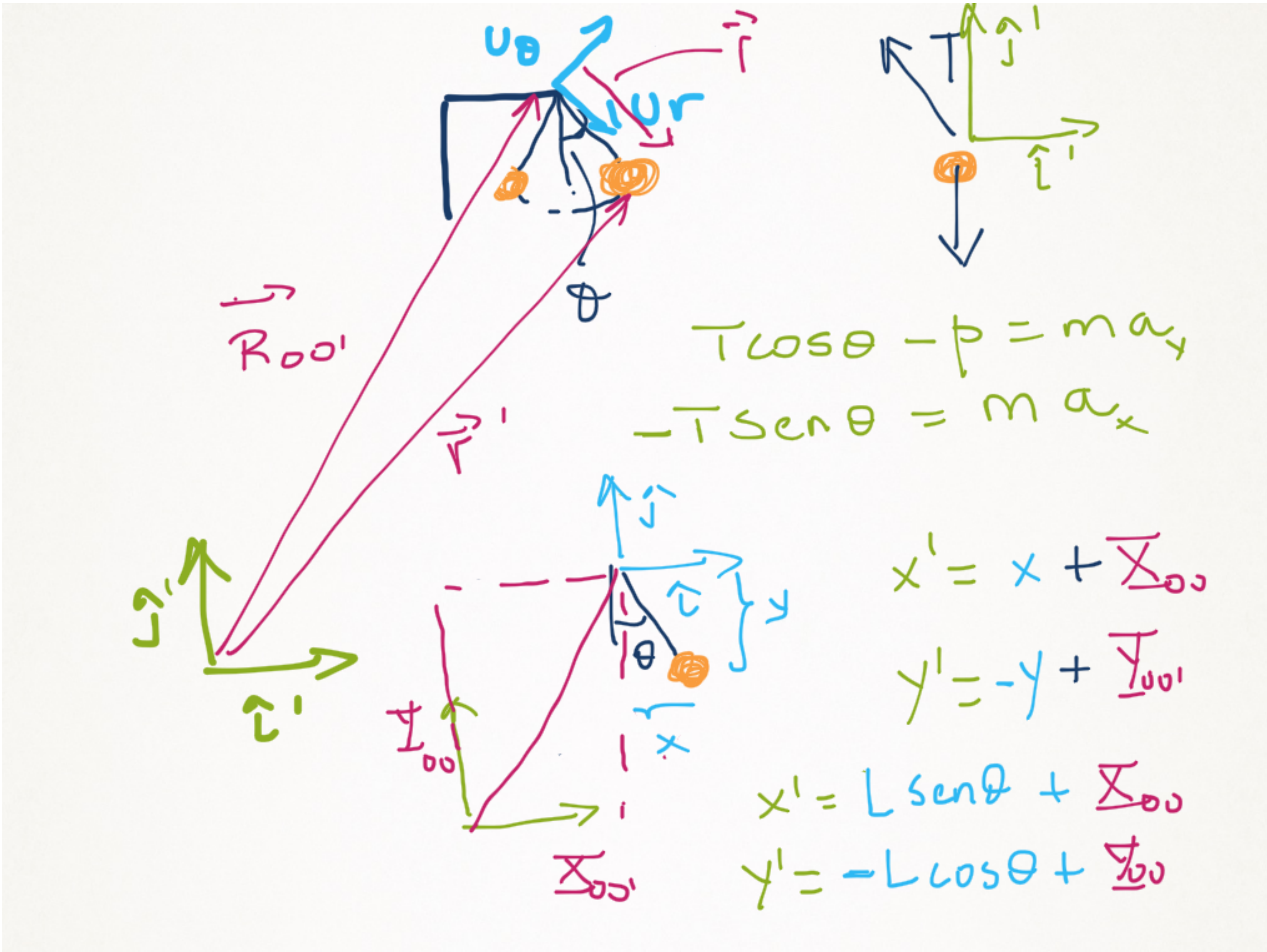
Traslacionales



Traslacionales: Equilibrio



E pur si muove (1) ....



E pur si muove (2)....

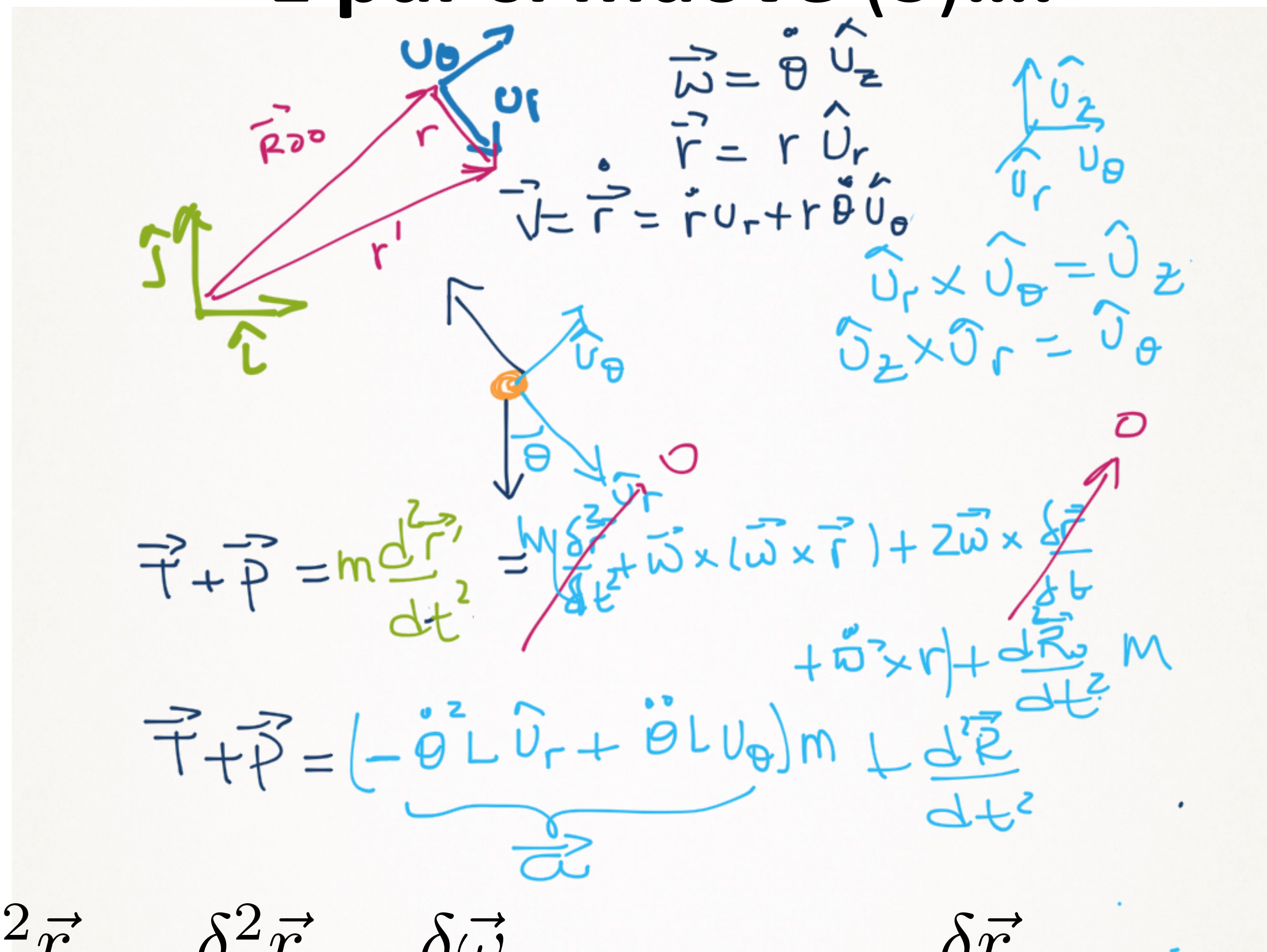
$$x' = L\,\mathrm{sen}\,\theta + X_{00'}$$
$$y' = -L\cos\theta + Y_{00'}$$

$$V'_x = L\dot{\theta}\cos\theta + V_{00'x}$$
$$V'_y = L\dot{\theta}\,\mathrm{sen}\,\theta + V_{00'y}$$

$$a'_x = L\ddot{\theta}\cos\theta - L\dot{\theta}^2\,\mathrm{sen}\,\theta + a_{00'x}$$
$$a'_y = L\ddot{\theta}\,\mathrm{sen}\,\theta + L\dot{\theta}^2\cos\theta + a_{00'y}$$

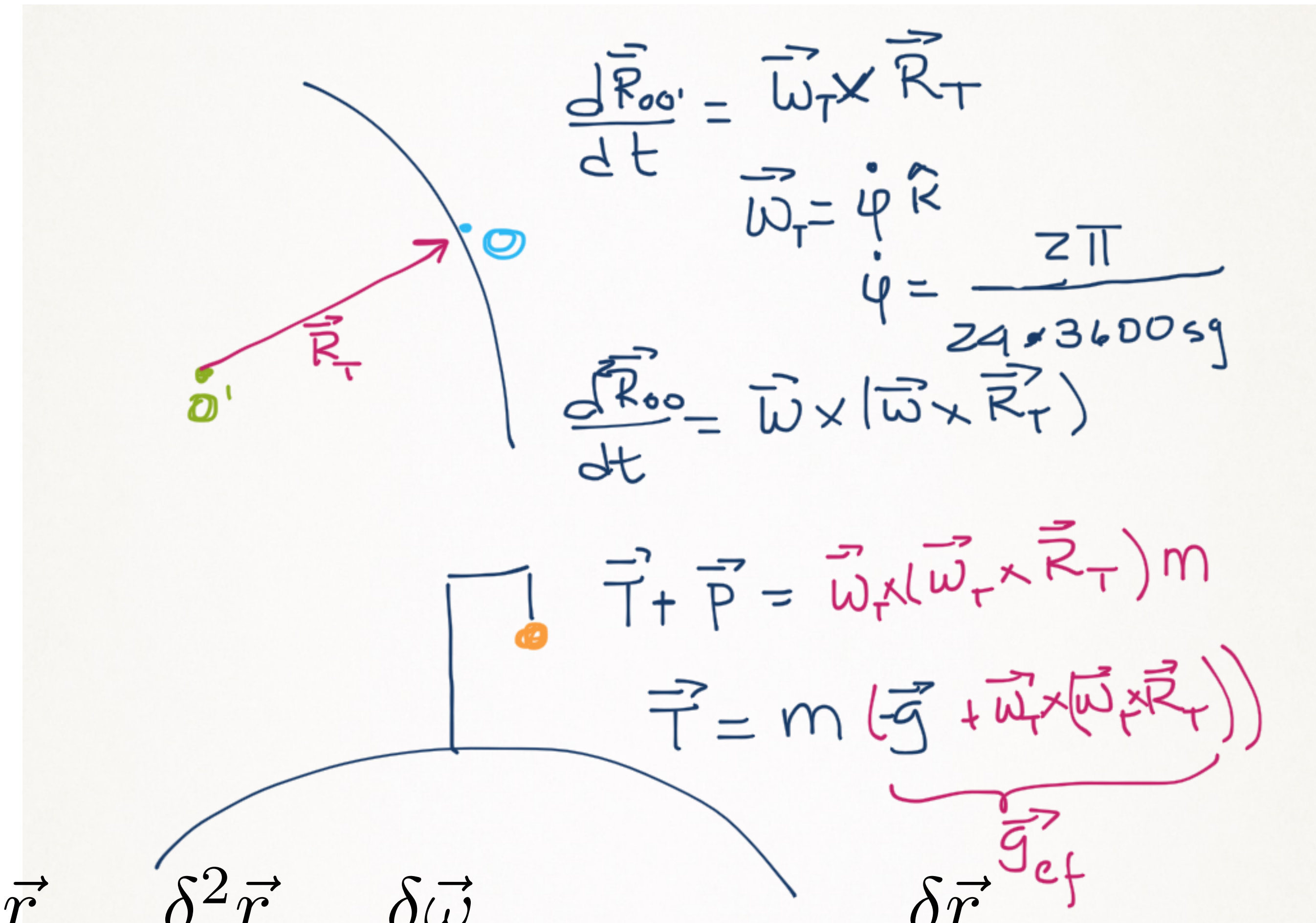
$$T\cos\theta - P = mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\,\mathrm{sen}\,\theta + ma_{0x}$$
$$-T\,\mathrm{sen}\,\theta = mL\ddot{\theta}\,\mathrm{sen}\,\theta + L\dot{\theta}^2\cos\theta + a_{00y}\,m$$

E pur si muove (3)....



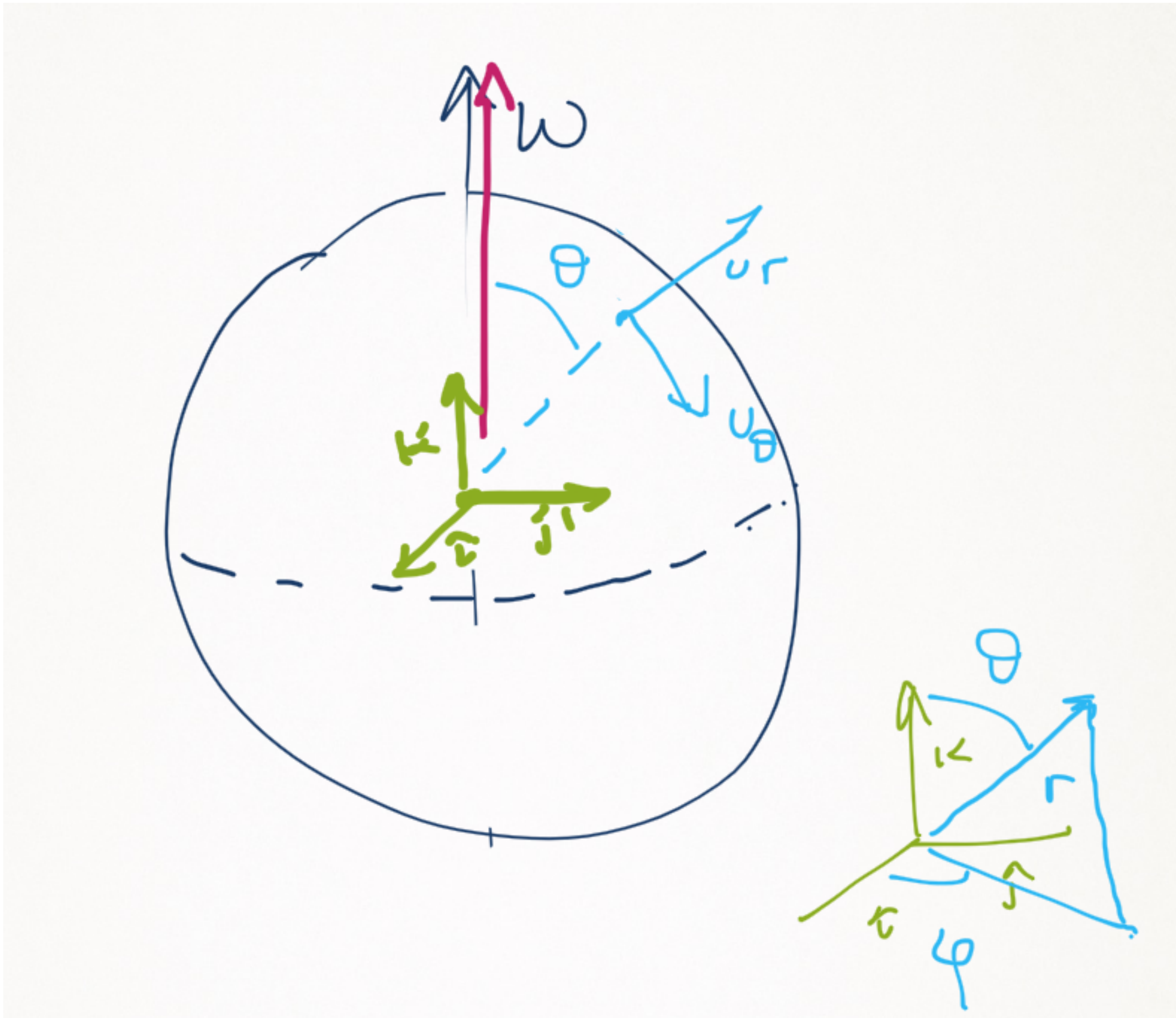
$$\vec{a} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\delta^2\vec{r}}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + 2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

E pur si muove (3)....

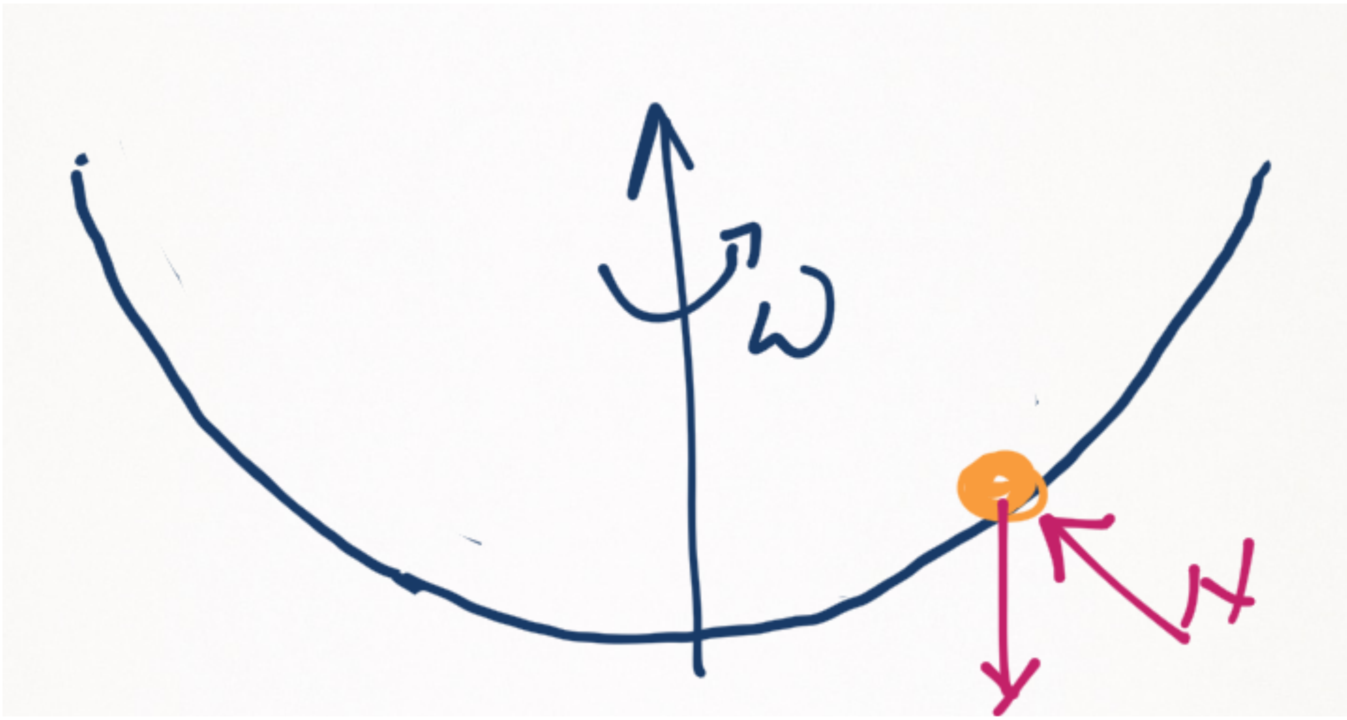


$$\vec{a} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\delta^2\vec{r}}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + 2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

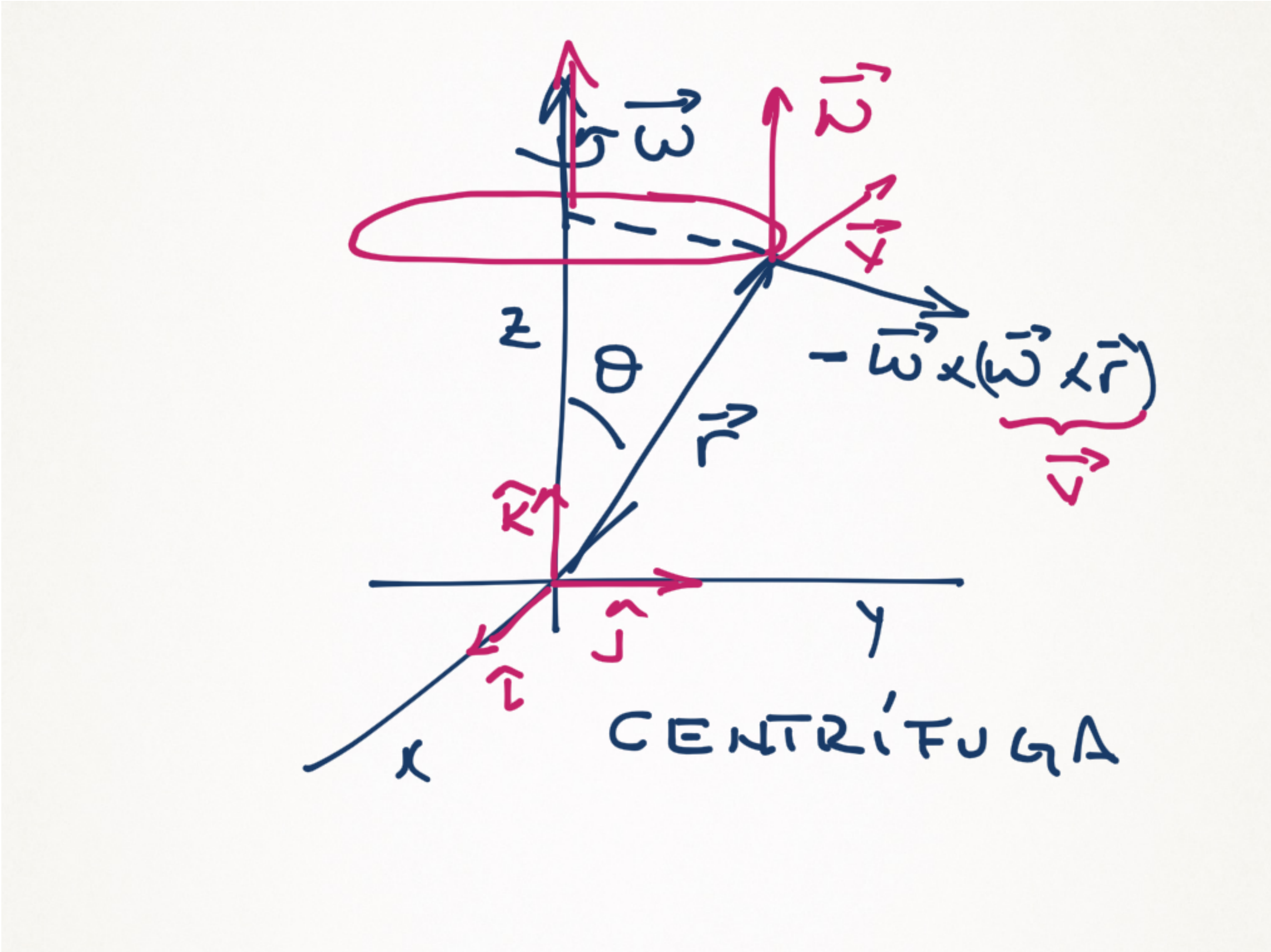
Coordenadas polares, cilíndricas y esféricas



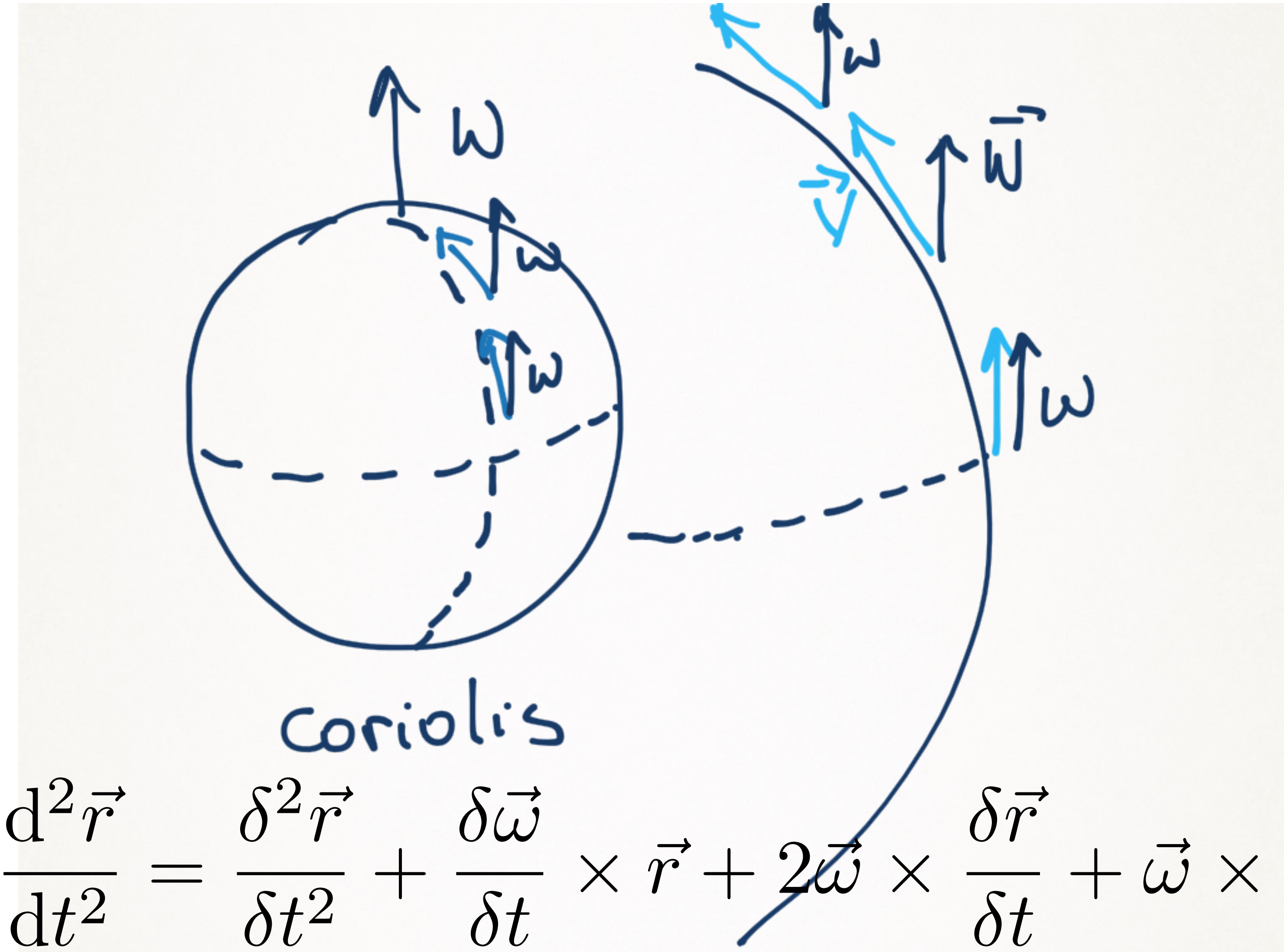
Centrífuga



¿Centrífuga?



Coriolis



$$\vec{a} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{\delta^2\vec{r}}{\delta t^2} + \frac{\delta\vec{\omega}}{\delta t} \times \vec{r} + 2\vec{\omega} \times \frac{\delta\vec{r}}{\delta t} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Péndulo Foucault