Sistemas de Computación Algebraica como Ambientes de Cálculo Científico

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Algebra Computacional ?

En el campo de la computación científica los métodos y herramientas de análisis numérico son tradicionalmente más comunes que los **Sistemas de Computación Algebraica** (**SCA**).

La expresión: "*cálculo por computadora* " generalmente se entiende como Computación Numérica

- FORTRAN, C, C++
- Precisión fija (Punto Flotante)

Los **SCA** son programas que pueden operar con ***símbolos*** que representan ***Objetos Matemáticos***:

- Números (Enteros, racionales, reales, complejos...)
- Polinómios, Funciones Racionales, Sistemas

de Ecuaciones,

- Grupos, Anillos, Algebras . . .

Estas herramientas trabajan de la forma:

Input: solve(problema);
Output: respuesta

- **Sistemas de Propósito Especial:**

  FORM, GAP, CAMAL, SHEEP
  STENSOR, LiE, KANT

- **Sistemas de Propósito General**

  Axiom, Derive, Reduce, Maple V, MatLab,
  Mathematica, Macsyma, MuPAD, GAUSS

Propiedades de los SCA

- Son programas Interactivos
- Precisión arimética arbitraria
- Cálculos exactos con expresiones matemáticas simbólicas
- Manipulación de expresiones y subexpresiones
- Aproximación analítica y numérica
- Extensión a la programación

Sistemas de Computación Algebraica Modernos de Propósito General

Un **SCA** de propósito general es un ambiente completamente integrado de computación para la investigación y la educación:

- Interfaz gráfica (worksheet) o ambiente interactivo:

  * Procesador de texto, de fórmulas y de gráficas.

* Con salidas en Latex, RTF, HTML, FORTRAN y C; o hyperlinks a otros documentos.
* Manuales en línea.
* Enlaces a otros programas y bibliotecas

- Capacidades para cálculo numérico
- Capacidades para visualización, con salidas gráficas en diferentes formatos: PostScript, GIF,JPG, . . .
- Pensado para usuarios no especializados en computación.

Los más populares **SCA** :

Derive	3.0
Macsyma	2.4
Reduce	3.6
Mathematica	4.0
Axiom	2.0
Maple V	5.1
MuPAD	1.4
GAUSS	3.2
Matlab	5.3

Principal Ventaja:

Enorme capacidad para realizar cálculos algebraicos largos y tediosos.

Por ejemplo, demostrar que la función:

es solución de la Ecuación Diferencial:

Puede tomarle a un PC un tiempo de CPU relativamente corto:

tiempo de cpu = 5,365 seg

Un Ejemplo: Maple V

MapleV es una herramienta de computación científica con las siguientes caracteristicas generales:

- ∞ Manipulador Simbólico
- ∞ Gran colección de Funciones Numéricas
- ∞ Capacidad gráfica en 2D y 3D
- ∞ Lenguaje de programación Avanzada
- ∞ Sintaxis similar al FORTRAN, PASCAL o C
- ∞ Hoja de Cálculo y Editor de Texto con salidas en Latex y HTML

Plataformas:

- ∞ **Maple V** está disponible para DOS, Windows, MacOS, UNIX
- ∞ La gran ventaja: una hoja de cálculo puede ser utilizada sobre cualquier plataforma sin necesidad de ser alterada.

1 Sintaxis Básica:

1.- Zona de Texto (En color Negro)
2.- Zona de comandos (En Color Rojo) (Cada comando debe finalizar con **;** o con :)
3.- Zona de respuestas (En color Azul)

```
> 25! + 13^23;
```

572651154567441023510457977 **(3.1)**

Las líneas con comentarios se colocan a continuación del símbolo #

```
> c^2  =  a^2 + b^2 ; # Teorema de Pitágoras
```

$$c^2 = a^2 + b^2 \tag{3.2}$$

El siguiente comando reinicia la hoja de cálculo

```
> restart;
```

2 Primeros Pasos

Maple V hace cálculos tanto con numeros enteros como en Punto Flotante

```
> 15 + 5^20;
```

$$95367431640640 \tag{4.1}$$

```
> 15. + 5^20;
```

$$9.536743164\ 10^{13} \tag{4.2}$$

Pero el énfasis radica en los cálculos exáctos

```
> cos(Pi/12)^2 + ln(2/3+5)/7;
```

$$\cos\left(\frac{1}{12}\pi\right)^2 + \frac{1}{7}\ln\left(\frac{17}{3}\right) \tag{4.3}$$

```
> evalf(%); # EVALuate using Floating-point
```

$$1.180812853 \tag{4.4}$$

Con % se puede utilizar la última salida de **Maple V** como entrada en el comando siguiente.

Existe un orden de precedencia para los operadores: **+ - * / ^**

```
> 1+2*3^2;
```

$$19 \tag{4.5}$$

```
> (1+2)*3^2;
```

$$27 \tag{4.6}$$

Por lo tanto, su uso descuidado produce errores

```
> 2^3^5;
Error, ambiguous use of `^`, please use parentheses
> 2^-5;
Error, `-` unexpected
```

Constantes numéricas y letras griegas. **MapleV** hace distinción entre mayúsculas y minúsculas

```
> evalf(Pi);
```

$$3.141592654 \tag{4.7}$$

```
> evalf(pi);
```

$$\pi \tag{4.8}$$

La precisión de su evaluación puede ser controlada:

```
> sqrt( 68 ); sqrt( 68. ); evalf(sqrt(68),50 );
```

$$2\sqrt{17}$$

$$8.246211251$$

$$8.2462112512353210996428197119481540502943984507472 \tag{4.9}$$

Se puede redondear a una fracción

```
> convert(%,fraction);
```

$$\frac{143649}{17420} \tag{4.10}$$

```
> restart;
```

Para asignar un objeto a una variable se hace con el operador :=

```
> f := arctan( (2*x^2-1)/(2*x^2+1) );
```

$$f := \arctan\left(\frac{2x^2-1}{2x^2+1}\right) \tag{4.11}$$

```
> f;
```

$$\arctan\left(\frac{2x^2-1}{2x^2+1}\right) \tag{4.12}$$

Mientras que el simbolo = , se utliza para mostrar una relación entre variables

```
> h = x + y;
```

$$h = x + y \tag{4.13}$$

```
> h;
```

$$h \qquad (4.14)$$

Por lo tanto, existen nombres que ya estan definidos

```
> abs:= f*sin(x);
Error, attempting to assign to `abs` which is protected
```

abs es la función valor absoluto

```
> abs(x-a);
```

$$|-x+a| \qquad (4.15)$$

```
> restart;
```

3 Cálculo Elemental

Se definen, se evalúan y se derivan funciones abstractas

```
> f:= (x,y) -> exp(x^2+y^2)/(x-y);
```

$$f := (x, y) \to \frac{e^{x^2+y^2}}{x-y} \qquad (5.1)$$

```
> f(a^(1/2),b^(1/2));
```

$$\frac{e^{a+b}}{\sqrt{a}-\sqrt{b}} \qquad (5.2)$$

```
> f(2,3);
```

$$-e^{13} \qquad (5.3)$$

```
> evalf(%);
```

$$-4.424133920\ 10^{5} \qquad (5.4)$$

```
> diff( f(x,y) , x) + diff( f(x,y) , y)
;
```

$$\frac{2\,x\,e^{x^2+y^2}}{x-y} + \frac{2\,y\,e^{x^2+y^2}}{x-y} \qquad (5.5)$$

Conserva las expresiones para la derivada interna

```
> f:= (t,r) -> g(c*t+r) + h(c*t-r);
```

$$f := (t, r) \to g(c\,t+r) + h(c\,t-r) \qquad (5.6)$$

```
> diff(f(t,r),t$3);
```

$$D^{(3)}(g)(c\,t+r)\,c^3 + D^{(3)}(h)(c\,t-r)\,c^3 \qquad (5.7)$$

Para funciones compuestas se utiliza el operador @ y para derivar el operador **D**

```
> (cos@ln@sqrt)(x);
```

$$\cos\left(\frac{1}{2}\ln(x)\right) \quad (5.8)$$

```
> D(cos@ln@sqrt)(x);
```

$$-\frac{1}{2}\,\frac{\sin\left(\frac{1}{2}\ln(x)\right)}{x} \quad (5.9)$$

```
> diff(cos(ln(sqrt(x))),x);
```

$$-\frac{1}{2}\,\frac{\sin\left(\frac{1}{2}\ln(x)\right)}{x} \quad (5.10)$$

Límites:

```
> f:= (x) -> 1/(1+r/x)^x;
```

$$f := x \rightarrow \frac{1}{\left(1+\frac{r}{x}\right)^{x}} \quad (5.11)$$

```
> Limit(f(x),x=infinity) = limit(f(x),
  x=infinity);
```

$$\lim_{x \rightarrow \infty} \frac{1}{\left(1+\frac{r}{x}\right)^{x}} = e^{-r} \quad (5.12)$$

Las mayúsculas a la izquierda (**Limit**) proveen una representación del Límite. Las minúsculas (**limit**) su ejecución.

También podemos calcular límites a la izquierda y a la derecha.

```
> g:= (x)-> tan(x + Pi) ;
```

$$g := x \rightarrow \tan(x+\pi) \quad (5.13)$$

```
> Limit(g(x), x=Pi/2,right)=limit(g(x),
  x=Pi/2,right);
```

$$\lim_{x \rightarrow \frac{1}{2}\pi^{+}} \tan(x) = -\infty \quad (5.14)$$

Sumatorias:

```
> Sum((1+i)/i^2, i = 1..n) ;
```

$$\sum_{i=1}^{n} \frac{1+i}{i^2} \tag{5.15}$$

```
> value(%);
```

$$-\Psi(1, n+1) + \Psi(n+1) + \frac{1}{6}\pi^2 + \gamma \tag{5.16}$$

```
> ?gamma
> ?Psi
```

Integración:

```
> g:=(x)-> 1/( x*( b*x+c*x^2)^(1/2) );
```

$$g := x \to \frac{1}{x\sqrt{b\,x + c\,x^2}} \tag{5.17}$$

```
> Int( g(x) , x ): % =  value(%);
```

$$\int \frac{1}{x\sqrt{b\,x + c\,x^2}}\,dx = -\frac{2\,(b + c\,x)}{b\sqrt{b\,x + c\,x^2}} \tag{5.18}$$

Evalua integrales definidas tanto analítica como numéricamente

```
> f:=(x)->  exp(Pi*x);
```

$$f := x \to e^{\pi x} \tag{5.19}$$

```
> Int( f(x) , x=1..3): %= value(%);
```

$$\int_{1}^{3} e^{\pi x}\,dx = \frac{e^{\pi}\left(-1 + e^{2\pi}\right)}{\pi} \tag{5.20}$$

```
> Int( f(x) , x=1..3 ): %= evalf(%);
```

$$\int_{1}^{3} e^{\pi x}\,dx = 3937.018092 \tag{5.21}$$

Evalua integrales impropias y las funciones que de ella emergen

```
> h:= (t)-> exp(-t)*t^(z-1);
```

$$h := t \to e^{-t}\,t^{z-1} \tag{5.22}$$

```
> Int(h(t),t=0..infinity): %= value(%);
```

(5.23)

$$\int_0^\infty e^{-t}\, t^{z-1}\, dt = \Gamma(z) \quad (5.23)$$

```
> assume(z>0):
> Int(h(t),t=0..infinity): %= value(%);
```

$$\int_0^\infty e^{-t}\, t^{z\sim-1}\, dt = \Gamma(z\sim) \quad (5.24)$$

```
> ?GAMMA
> GAMMA(0.5); # Funcion Gamma
```

$$1.772453851 \quad (5.25)$$

```
> Int(x/(x^2+2*x+2),x): %= value(%);
```

$$\int \frac{x}{x^2+2x+2}\, dx = \frac{1}{2}\ln\left(x^2+2x+2\right) - \arctan(x+1) \quad (5.26)$$

```
> diff(rhs(%),x);# Chequeamos que es la
  solucion correcta
```

$$\frac{1}{2}\frac{2x+2}{x^2+2x+2} - \frac{1}{1+(x+1)^2} \quad (5.27)$$

```
> simplify(%);
```

$$\frac{x}{x^2+2x+2} \quad (5.28)$$

```
> restart;
```

4 Resolución de sistemas de ecuaciones algebraicas

Resuelve sistemas de ecuaciones algebráicas. El resultado queda almacenado en una lista, y luego hay que asignar esos valores al "nombre" de las variables correspondientes para seguir operando.

```
> ecu1:= R[1]*i[1]+R[3]*i[3]+R[4]*i[4]-V
  [1]=0;
```

$$ecu1 := R_1 i_1 + R_3 i_3 + R_4 i_4 - V_1 = 0 \quad (6.1)$$

```
> ecu2:= R[2]*i[2]-V[2]-R[3]*i[3]=0;
```

$$ecu2 := R_2 i_2 - V_2 - R_3 i_3 = 0 \quad (6.2)$$

```
> ecu3:=  i[1]-i[2]-i[3]= 0;
```

$$ecu3 := i_1 - i_2 - i_3 = 0 \quad (6.3)$$

```
> solve({ecu1,ecu2,ecu3},{i[1],i[2],i[3]
  } ); assign(%);
```

$$\left\{i_1 = -\frac{-V_2 R_3 + R_3 R_4 i_4 - R_3 V_1 + R_2 R_4 i_4 - R_2 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3}, i_2 = \frac{R_1 V_2 + V_2 R_3 - R_3 R_4 i_4 + R_3 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3}, i_3 = -\frac{R_1 V_2 + R_2 R_4 i_4 - R_2 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3}\right\} \quad \textbf{(6.4)}$$

```
> i[1],i[2],i[3];
```

$$-\frac{-V_2 R_3 + R_3 R_4 i_4 - R_3 V_1 + R_2 R_4 i_4 - R_2 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3}, \frac{R_1 V_2 + V_2 R_3 - R_3 R_4 i_4 + R_3 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3}, -\frac{R_1 V_2 + R_2 R_4 i_4 - R_2 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3} \quad \textbf{(6.5)}$$

Ahora al evaluar x, y, z en las ecuaciones ecu1 ... ecu3

```
> ecu1; ecu2; ecu3;
```

$$-\frac{R_1 \left(-V_2 R_3 + R_3 R_4 i_4 - R_3 V_1 + R_2 R_4 i_4 - R_2 V_1\right)}{R_2 R_1 + R_2 R_3 + R_1 R_3} - \frac{R_3 \left(R_1 V_2 + R_2 R_4 i_4 - R_2 V_1\right)}{R_2 R_1 + R_2 R_3 + R_1 R_3} + R_4 i_4 - V_1 = 0$$

$$\frac{R_2 \left(R_1 V_2 + V_2 R_3 - R_3 R_4 i_4 + R_3 V_1\right)}{R_2 R_1 + R_2 R_3 + R_1 R_3} - V_2 + \frac{R_3 \left(R_1 V_2 + R_2 R_4 i_4 - R_2 V_1\right)}{R_2 R_1 + R_2 R_3 + R_1 R_3} = 0$$

$$-\frac{-V_2 R_3 + R_3 R_4 i_4 - R_3 V_1 + R_2 R_4 i_4 - R_2 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3} - \frac{R_1 V_2 + V_2 R_3 - R_3 R_4 i_4 + R_3 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3} + \frac{R_1 V_2 + R_2 R_4 i_4 - R_2 V_1}{R_2 R_1 + R_2 R_3 + R_1 R_3} = 0 \quad \textbf{(6.6)}$$

```
> simplify(ecu1); simplify(ecu2);
  simplify(ecu3);
```

$$0 = 0$$

$$0 = 0$$

$$0 = 0 \quad \textbf{(6.7)}$$

Seguidamente procedemos a "limpiar" el contenido de las variables para poder utilizarlas en otras expresiones.

```
> i[1]:='i[1]';  i[2]:='i[2]';  i[3]:='i
```

```
[3]';
```

$$i_1 := i_1$$

$$i_2 := i_2$$

$$i_3 := i_3 \qquad (6.8)$$

Soluciones aproximadas para encontrar las raíces de un polinomio

```
> p:= x^3 - 3*x^2 = 17*x - 51;
```

$$p := x^3 - 3\,x^2 = 17\,x - 51 \qquad (6.9)$$

```
> sol:=solve(p,x);
```

$$sol := 3, \sqrt{17}, -\sqrt{17} \qquad (6.10)$$

```
> evalf(%);
```

$$3., 4.123105626, -4.123105626 \qquad (6.11)$$

```
> q:= x^4 +3*x^3 - 6*x^2  + 12*x - 14=0;
```

$$q := x^4 + 3\,x^3 - 6\,x^2 + 12\,x - 14 = 0 \qquad (6.12)$$

```
> solve(q , x);
```

$$RootOf\left(\_Z^4 + 3\,\_Z^3 - 6\,\_Z^2 + 12\,\_Z - 14, index = 1\right), RootOf\left(\_Z^4 + 3\,\_Z^3 - 6\,\_Z^2 + 12\,\_Z - 14, index = 2\right), RootOf\left(\_Z^4 + 3\,\_Z^3 - 6\,\_Z^2 + 12\,\_Z - 14, index = 3\right), RootOf\left(\_Z^4 + 3\,\_Z^3 - 6\,\_Z^2 + 12\,\_Z - 14, index = 4\right) \qquad (6.13)$$

```
> q1 := RootOf(_Z^4+3*_Z^3-6*_Z^2+12*_Z-14, index = 1);
```

$$q1 := RootOf\left(\_Z^4 + 3\,\_Z^3 - 6\,\_Z^2 + 12\,\_Z - 14, index = 1\right) \qquad (6.14)$$

```
> allvalues(%);# evalua expresiones con RootOf
```

$$-\frac{3}{4} - \frac{1}{12}\sqrt{\frac{225\left(2025 + 15\sqrt{79665}\right)^{1/3} - 12\left(2025 + 15\sqrt{79665}\right)^{2/3} + 2880}{\left(2025 + 15\sqrt{79665}\right)^{1/3}}} + \frac{1}{12}\sqrt{6} \qquad (6.15)$$

$$\Bigg(\Bigg(75\left(2025 + 15\sqrt{79665}\right)^{1}$$

$$
{}^{/3}\sqrt{\frac{225\left(2025+15\sqrt{79665}\right)^{1/3}-12\left(2025+15\sqrt{79665}\right)^{2/3}+2880}{\left(2025+15\sqrt{79665}\right)^{1/3}}}
$$

$$
+2\sqrt{\frac{225\left(2025+15\sqrt{79665}\right)^{1/3}-12\left(2025+15\sqrt{79665}\right)^{2/3}+2880}{\left(2025+15\sqrt{79665}\right)^{1/3}}}
$$

$$
\left(2025+15\sqrt{79665}\right)^{2/3}
$$

$$
-480\sqrt{\frac{225\left(2025+15\sqrt{79665}\right)^{1/3}-12\left(2025+15\sqrt{79665}\right)^{2/3}+2880}{\left(2025+15\sqrt{79665}\right)^{1/3}}}
$$

$$
+1755\left(2025+15\sqrt{79665}\right)^{1/3}\Bigg)\Bigg/\Bigg(\left(2025+15\sqrt{79665}\right)^{1}
$$

$$
{}^{1/2}
$$

$$
{}^{/3}\sqrt{\frac{225\left(2025+15\sqrt{79665}\right)^{1/3}-12\left(2025+15\sqrt{79665}\right)^{2/3}+2880}{\left(2025+15\sqrt{79665}\right)^{1/3}}}\Bigg)\Bigg)
$$

```
> fsolve(q,x); # fsolve utiliza punto-flotante
```

$$-4.862978677,\ 1.253920186 \tag{6.16}$$

```
> restart:
```

5 Ecuaciones Diferenciales

```
> ED1:=diff(y(x),x)+y(x)/x=alpha/(x*y(x)^2);
```

$$ED1 := \frac{d}{dx}\,y(x)+\frac{y(x)}{x}=\frac{\alpha}{x\,y(x)^2} \tag{7.1}$$

```
> dsolve(ED1,y(x));
```

$$y(x)=\frac{\left(x^3\,\alpha+\_C1\right)^{1/3}}{x},\ y(x)=\frac{-\frac{1}{2}\left(x^3\,\alpha+\_C1\right)^{1/3}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(x^3\,\alpha+\_C1\right)^{1/3}}{x}, \tag{7.2}$$

$$y(x)=\frac{-\frac{1}{2}\left(x^3\,\alpha+\_C1\right)^{1/3}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(x^3\,\alpha+\_C1\right)^{1/3}}{x}$$

```
> sols:=dsolve({ED1,y(1)=5},y(x));
```

$$sols := y(x) = \frac{\left(x^3\,\alpha + 125 - \alpha\right)^{1/3}}{x} \tag{7.3}$$

Resuelve ecuaciones diferenciales, con y sin valores iniciales por varios métodos. Series . . .

```
> dsolve({ED1,y(1)=5},y(x),series);
```

$$y(x) = 5 + \left(-5 + \frac{1}{25}\,\alpha\right)(x-1) + \left(-\frac{1}{3125}\,\alpha^2 + 5\right)(x-1)^2 + \left(-\frac{1}{3125}\,\alpha^2 + \frac{1}{234375}\,\alpha^3 - 5 + \frac{1}{75}\,\alpha\right)(x-1)^3 + \left(\frac{2}{234375}\,\alpha^3 + 5 - \frac{1}{75}\,\alpha - \frac{2}{9375}\,\alpha^2 - \frac{2}{29296875}\,\alpha^4\right)(x-1)^4 + \left(\frac{2}{234375}\,\alpha^3 - \frac{2}{9765625}\,\alpha^4 - 5 + \frac{1}{75}\,\alpha + \frac{22}{18310546875}\,\alpha^5\right)(x-1)^5 + \mathrm{O}\left((x-1)^6\right) \tag{7.4}$$

Sistemas de Ecuaciones Difereciales. Mediante transformaciones de Laplace. . .

```
> sys:= diff(y(x),x)=z(x) , diff(z(x),x)
  =y(x);
```

$$sys := \frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = z(x),\ \frac{\mathrm{d}}{\mathrm{d}x}\,z(x) = y(x) \tag{7.5}$$

```
> fcns:={y(x),z(x)};
```

$$fcns := \{y(x), z(x)\} \tag{7.6}$$

```
> s:=dsolve({sys,y(0)=0,z(0)=1},fcns,
  method=laplace);
```

$$s := \{y(x) = \sinh(x), z(x) = \cosh(x)\} \tag{7.7}$$

```
> s[1]; # Selecciono una de las
  soluciones
```

$$y(x) = \sinh(x) \tag{7.8}$$

Puedo tomar únicamnte el lado derecho de la solución para una posterior manipulación

```
> 2*( rhs(s[1] )); # right hand sides
```

$$2\sinh(x) \tag{7.9}$$

Ecuaciones diferenciales famosas: **Bessel**

```
> ED2:=x^2*diff(y(x),x$2)+x*diff(y(x),x)
```

```
+(x^2-mu)*y(x)=0;
```

$$ED2 := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) + x\left(\frac{d}{dx}\, y(x)\right) + \left(x^2 - \mu\right) y(x) = 0 \tag{7.10}$$

```
> dsolve(ED2,y(x)); assign(%);
```

$$y(x) = \_C1\ \mathrm{BesselJ}\left(\sqrt{\mu}, x\right) + \_C2\ \mathrm{BesselY}\left(\sqrt{\mu}, x\right) \tag{7.11}$$

```
> simplify(ED2);
```

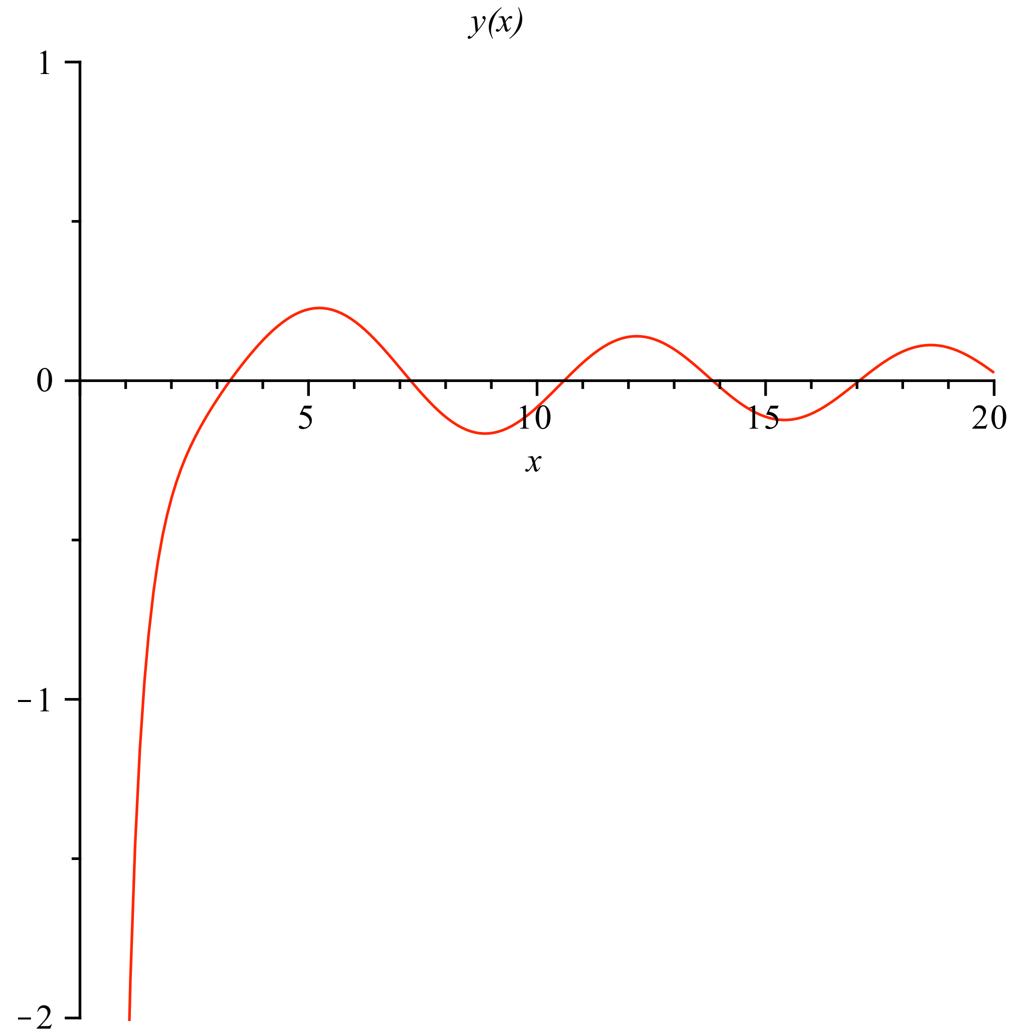
$$0 = 0 \tag{7.12}$$

Se pueden hacer graficas para un conjunto de condiciones iniciales

```
> y:=subs(mu=10, _C1=1/2, _C2=1/3 ,y(x))
  ;
```

$$y := \frac{1}{2}\ \mathrm{BesselJ}\left(\sqrt{10}, x\right) + \frac{1}{3}\ \mathrm{BesselY}\left(\sqrt{10}, x\right) \tag{7.13}$$

```
> plot(y, x=0..20,-2..1 ,title=`y(x)`);
```



Soluciones Numéricas:

```
> del :={diff(g(t),t$2)=-f(t)-g(t),diff
  (f(t),t$2)= diff(g(t),t)+f(t)};
```

$$del := \left\{ \frac{d^2}{dt^2} f(t) = \frac{d}{dt} g(t) + f(t),\ \frac{d^2}{dt^2} g(t) = -f(t) - g(t) \right\} \quad (7.14)$$

```
> init1 := {g(0)=1, D(g)(0)=0, f(0)=0, D
  (f)(0)=1}:
> F := dsolve(del union init1, {g(t),f
  (t)},type=numeric);
```

$$F := \textbf{proc}(x\_rkf45)\ \dots\ \textbf{end proc} \quad (7.15)$$

```
> F(0.0);
```

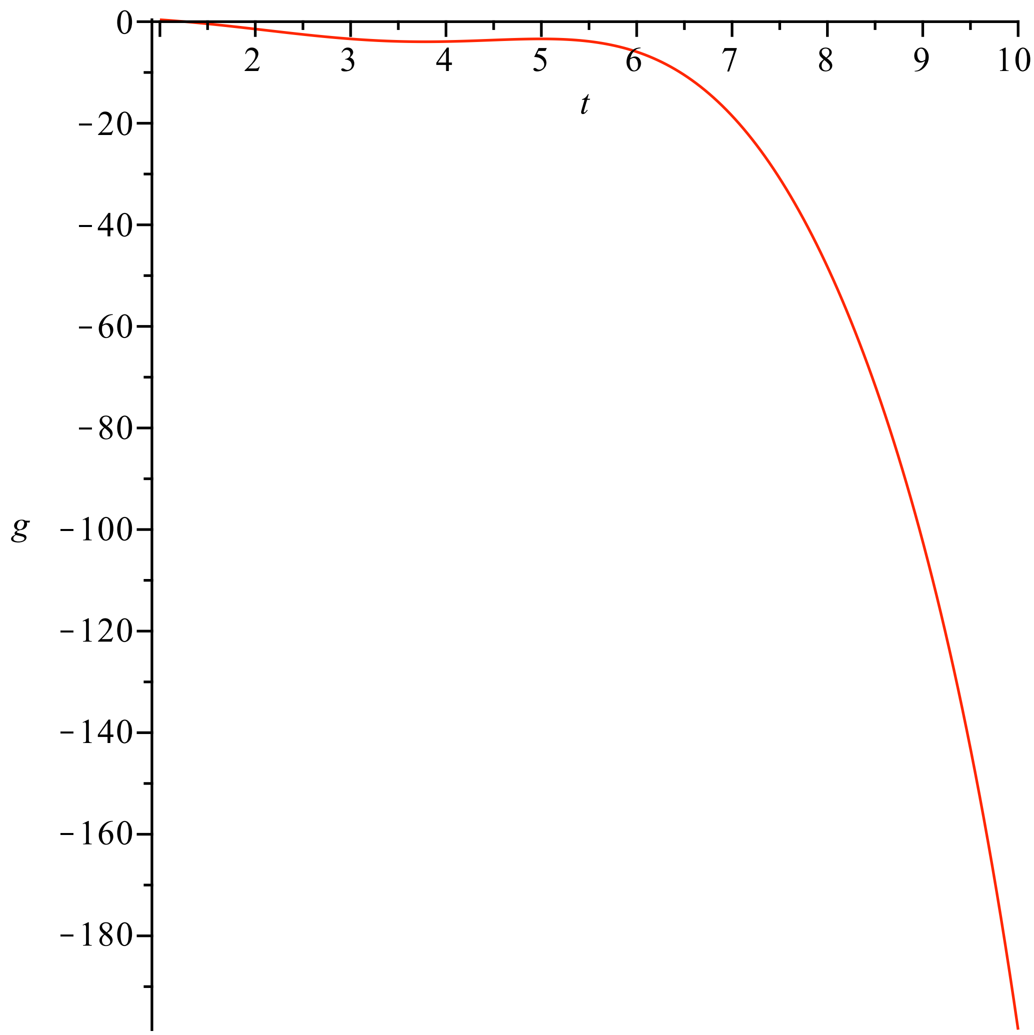
$$\left[t = 0.,\ f(t) = 0.,\ \frac{d}{dt} f(t) = 1.,\ g(t) = 1.,\ \frac{d}{dt} g(t) = 0. \right] \quad (7.16)$$

```
> F := dsolve(del union init1, {g(t),f
```

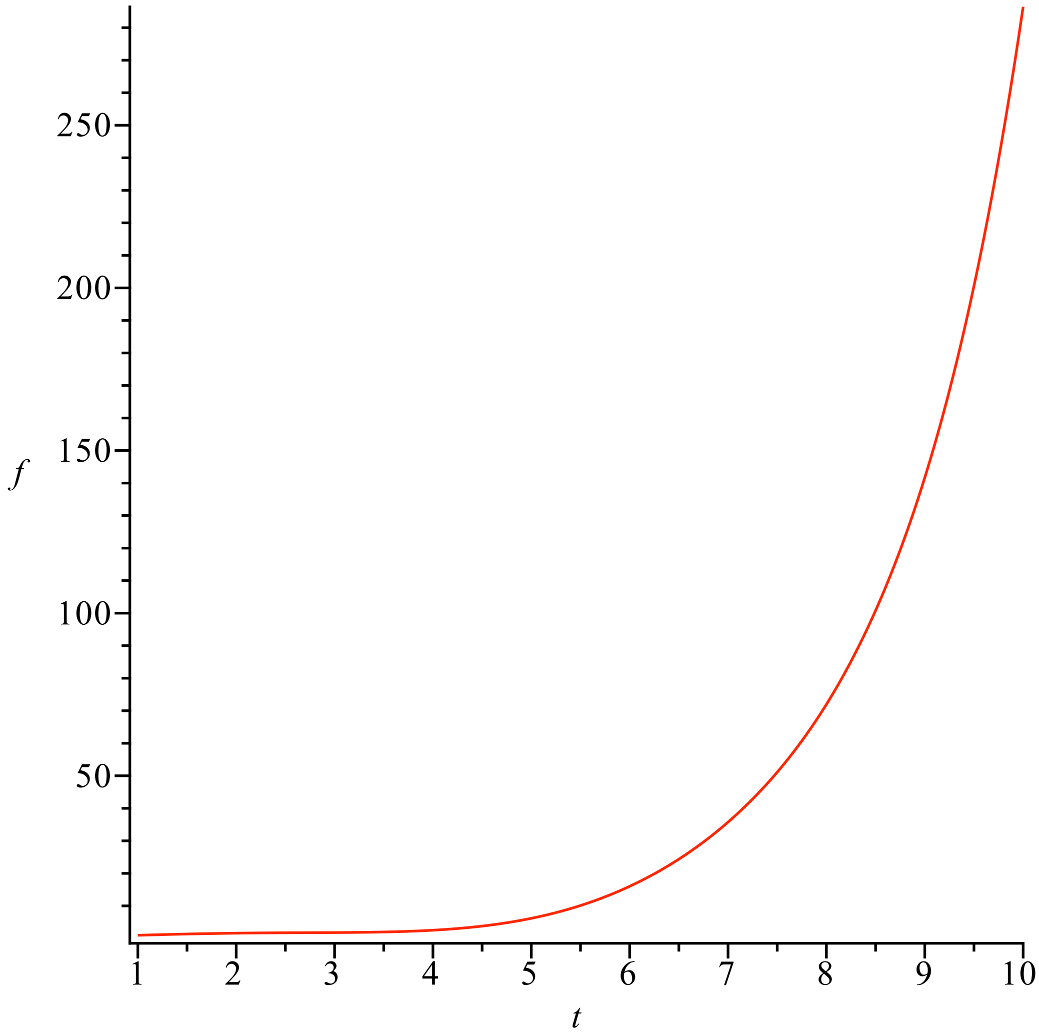
```
(t)},type=numeric, method=rosenbrock);
```

$F := \mathbf{proc}(x\_rosenbrock)\ \ldots\ \mathbf{end\ proc}$ **(7.17)**

```
> with(plots):
> odeplot(F,[t,g(t)],1..10,labels=[t,g])
;
```



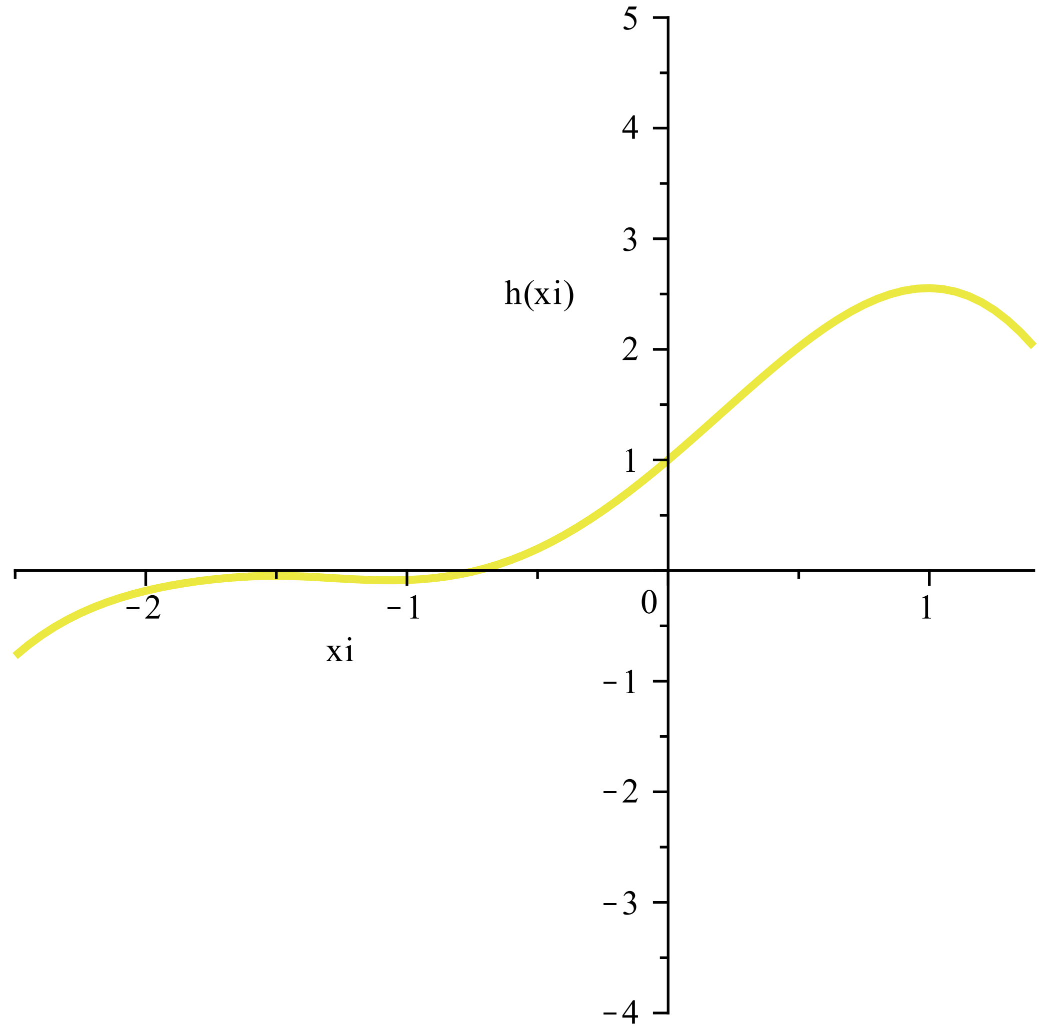

```
> odeplot(F,[t,f(t)],1..10,labels=[t,f])
;
```



```
> with(DEtools):
> S:=cos(xi)*diff(h(xi),xi$3)-diff(h(xi),xi$2)+Pi*diff(h(xi),xi)=h(xi)-xi;
```

$$S := \cos(\xi)\left(\frac{d^3}{d\xi^3} h(\xi)\right) - \left(\frac{d^2}{d\xi^2} h(\xi)\right) + \pi\left(\frac{d}{d\xi} h(\xi)\right) = h(\xi) - \xi \qquad (7.18)$$

```
> DEplot(S,h(xi),xi=-2.5..1.4,[[h(0)=1,D(h)(0)=2,(D@@2)(h)(0)=1]],h=-4..5,stepsize=.05);
```




```
> restart;
```

6 Manipulación algebraica

Maple V, automáticamente evalua y simplifica expresiones

```
> p:=(a+b)^3*(a+b)^2;
```

$$p := (a+b)^5 \tag{8.1}$$

```
> expand(p);
```

$$a^5 + 5\,a^4\,b + 10\,a^3\,b^2 + 10\,a^2\,b^3 + 5\,a\,b^4 + b^5 \tag{8.2}$$

```
> factor(p);
```

$$(a+b)^5 \tag{8.3}$$

```
> t:=cos(x)^5+sin(x)^4+2*cos(x)^2-2*sin(x)^2-cos(2*x);
```

$$t := \cos(x)^5 + \sin(x)^4 + 2\cos(x)^2 - 2\sin(x)^2 - \cos(2\,x) \tag{8.4}$$

```
> simplify(t);
```

$$\cos(x)^4\,(\cos(x)+1) \tag{8.5}$$

```
> r:=alpha*(x^3-y^3)/(beta*(x^2+x-y-y^2));
```

$$r := \frac{\alpha\,(x^3-y^3)}{\beta\,(x^2+x-y-y^2)} \tag{8.6}$$

```
> normal(r);
```

$$\frac{(x^2+y\,x+y^2)\,\alpha}{(x+1+y)\,\beta} \tag{8.7}$$

```
> n:=numer(r); d:=denom(r);
```

$$n := \alpha\,(x^3-y^3)$$

$$d := \beta\,(x^2+x-y-y^2) \tag{8.8}$$

```
> n*d*delta;
```

$$\alpha\,(x^3-y^3)\,\beta\,(x^2+x-y-y^2)\,\delta \tag{8.9}$$

```
> expand(%);
```

$$\alpha\,\beta\,\delta\,x^5+\alpha\,\beta\,\delta\,x^4-\alpha\,\beta\,\delta\,x^3\,y-\alpha\,\beta\,\delta\,x^3\,y^2-\alpha\,\beta\,\delta\,y^3\,x^2-\alpha\,\beta\,\delta\,y^3\,x+\alpha\,\beta\,\delta\,y^4+\alpha\,\beta\,\delta\,y^5 \tag{8.10}$$

```
> collect(%,x);
```

$$\alpha\,\beta\,\delta\,x^5+\alpha\,\beta\,\delta\,x^4+(-\alpha\,\beta\,\delta\,y-\alpha\,\beta\,\delta\,y^2)\,x^3-\alpha\,\beta\,\delta\,y^3\,x^2-\alpha\,\beta\,\delta\,y^3\,x+\alpha\,\beta\,\delta\,y^4+\alpha\,\beta\,\delta\,y^5 \tag{8.11}$$

```
> coeff(%,x^3);
```

$$-\alpha\,\beta\,\delta\,y-\alpha\,\beta\,\delta\,y^2 \tag{8.12}$$

```
> factor(%);
```

$$-\alpha\,\beta\,\delta\,y\,(1+y) \tag{8.13}$$

Puede convertir muchos tipos de expresiones en otras más especificas

```
> expr1:=(a*x^2+b)/(x*(-3*x^2-x+4));
```

$$expr1 := \frac{a\,x^2+b}{x\,(-3\,x^2-x+4)} \tag{8.14}$$

```
> expr2:=convert(expr1,parfrac,x);
```

$$expr2 := \frac{1}{28}\,\frac{-16\,a-9\,b}{3\,x+4}+\frac{1}{4}\,\frac{b}{x}+\frac{1}{7}\,\frac{-a-b}{x-1} \tag{8.15}$$

```
> expr3:=sin(x)*cos(x);
```

$$expr3 := \sin(x)\cos(x) \quad (8.16)$$

```
> expr4:=convert(%,exp);
```

$$expr4 := -\frac{1}{2} I\left(e^{Ix} - e^{-Ix}\right)\left(\frac{1}{2}e^{Ix} + \frac{1}{2}e^{-Ix}\right) \quad (8.17)$$

Se pueden aislar los operandos de las expresiones

```
> op(3,expr2); op(3,expr4);
```

$$\frac{1}{7}\,\frac{-a-b}{x-1}$$

$$\frac{1}{2}e^{Ix} + \frac{1}{2}e^{-Ix} \quad (8.18)$$

```
> op(3,expr2)*op(3,expr4);
```

$$\frac{1}{7}\,\frac{(-a-b)\left(\frac{1}{2}e^{Ix} + \frac{1}{2}e^{-Ix}\right)}{x-1} \quad (8.19)$$

```
> simplify(%);
```

$$-\frac{1}{7}\,\frac{(a+b)\cos(x)}{x-1} \quad (8.20)$$

No siempre todo funciona de la manera correcta. La siguiente intregal es trivial, pero veamos lo que sucede con **Maple V**

```
> Int(2*x*(x^2+1)^24,x);
```

$$\int 2x\left(x^2+1\right)^{24} dx \quad (8.21)$$

```
> value(%);
```

$$x^2 + 12x^4 + \frac{1}{25}x^{50} + x^{48} + 12x^{46} + 92x^{44} + 506x^{42} + \frac{10626}{5}x^{40} + 7084x^{38} + 19228x^{36} + 43263x^{34} + 81719x^{32} + \frac{653752}{5}x^{30} + 178296x^{28} + 208012x^{26} + 208012x^{24} + 178296x^{22} + \frac{653752}{5}x^{20} + 81719x^{18} + 43263x^{16} + 19228x^{14} + 7084x^{12} + \frac{10626}{5}x^{10} + 506x^8 + 92x^6 \quad (8.22)$$

```
> factor(%);
```

$$\frac{1}{25}x^2\left(x^8 + 5x^6 + 10x^4 + 10x^2 + 5\right)\left(x^{40} + 20x^{38} + 190x^{36} + 1140x^{34} + 4845x^{32} \quad (8.23)\right.$$

$$+ 15505 x^{30} + 38775 x^{28} + 77625 x^{26} + 126425 x^{24} + 169325 x^{22} + 187760 x^{20} + 172975 x^{18} + 132450 x^{16} + 84075 x^{14} + 43975 x^{12} + 18760 x^{10} + 6425 x^{8} + 1725 x^{6} + 350 x^{4} + 50 x^{2} + 5)$$

```
> factor(%+1/25);
```

$$\frac{1}{25} \left(x^2 + 1\right)^{25} \qquad (8.24)$$

```
> restart;
```

7 Bibliotecas

No todos los comandos son cargados en la memoria cuando **Maple V** es iniciado. Sólo los comandos estandard son cargados automáticamente

```
> P:=  x^5-24*x^4+85*x^3-108*x^2+166*x-120 ;
```

$$P := x^5 - 24 x^4 + 85 x^3 - 108 x^2 + 166 x - 120 \qquad (9.1)$$

```
> factor(P);
```

$$(x - 20)\ (x - 1)\ (x - 3)\ \left(x^2 + 2\right) \qquad (9.2)$$

```
> realroot(P);
```

$$[[0, 2], [2, 4], [16, 32]] \qquad (9.3)$$

```
> readlib(realroot):
> realroot(P,1);
```

$$[[1, 1], [3, 3], [20, 20]] \qquad (9.4)$$

Las Bibliotecas de **Maple V**:

- La biblioteca estandard
- La biblioteca de misceláneos
- Paquetes
- La biblioteca de programas desarrollados por usuarios (Share Library)

```
> ?index,packages
> restart:
```

8 Graficos y Visualización

Una mayor capacidad grafica es obtenida con el paquete plots

```
> with(plots);
```

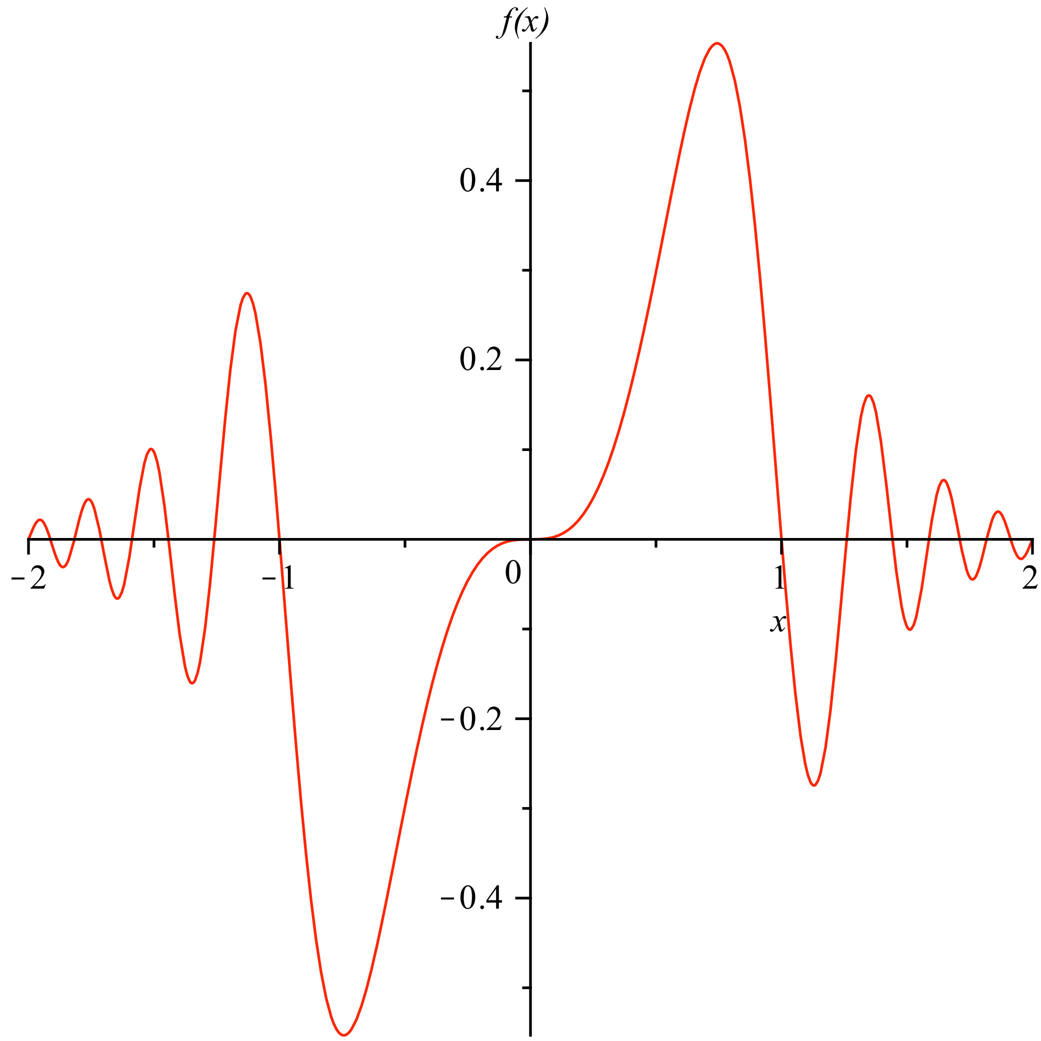
[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(10.1)**

Graficos básicos en 2 y 3 Dimensiones:

```
> f:=x-> exp(-x^2)*sin(Pi*x^3);
```

$$f := x \rightarrow e^{-x^2} \sin(\pi x^3) \quad \textbf{(10.2)}$$

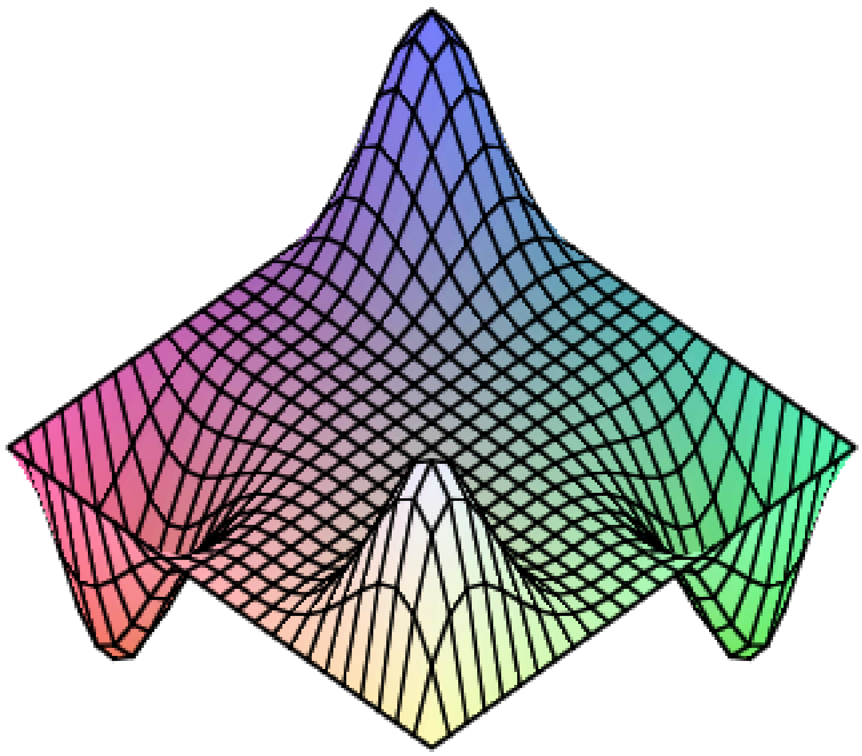
```
> plot( f(x),x=-2..2,title=`f(x)`);
```




```
> g:=(x,y)-> f(x)*f(y);
```

$$g := (x, y) \rightarrow f(x) f(y) \tag{10.3}$$

```
> plot3d(g(x,y),x=-1..1,y=-1..1);
```



```
> h:=(x) -> cos(sqrt(x^2+3*y^2))/(1+
  x^2/8);
```

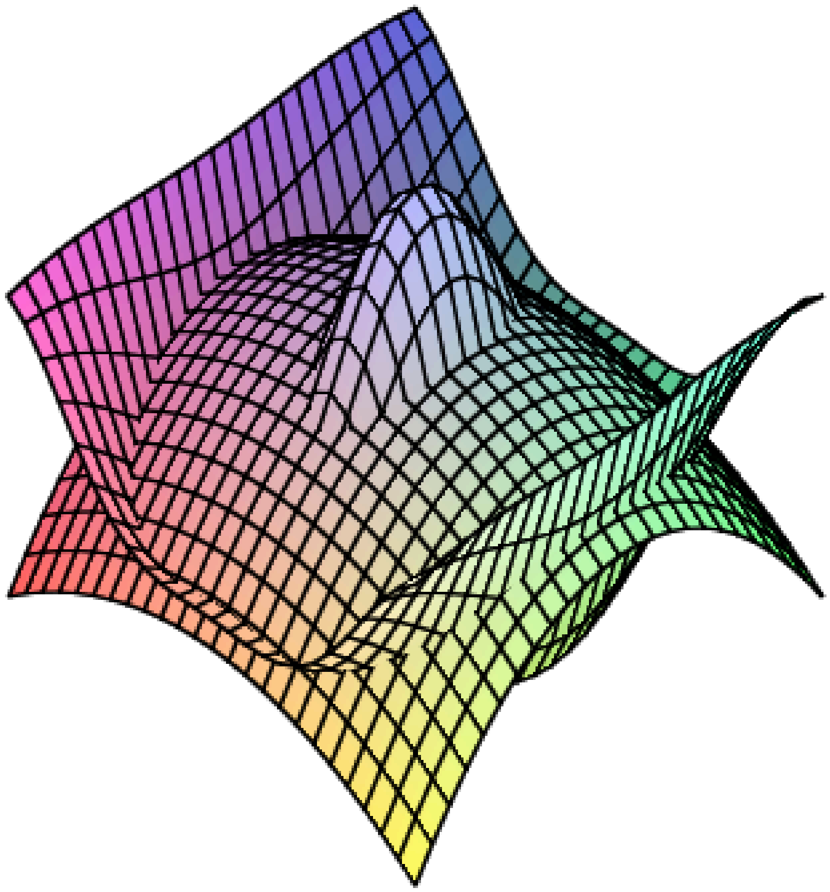
$$h := x \rightarrow \frac{\cos\left(\sqrt{x^2 + 3\,y^2}\right)}{1 + \frac{1}{8}\,x^2}$$

```
> j:= (x) -> 1/3 -(2*x^2+y^2)/19;
```

$$j := x \rightarrow \frac{1}{3} - \frac{2}{19}\,x^2 - \frac{1}{19}\,y^2 \tag{10.5}$$

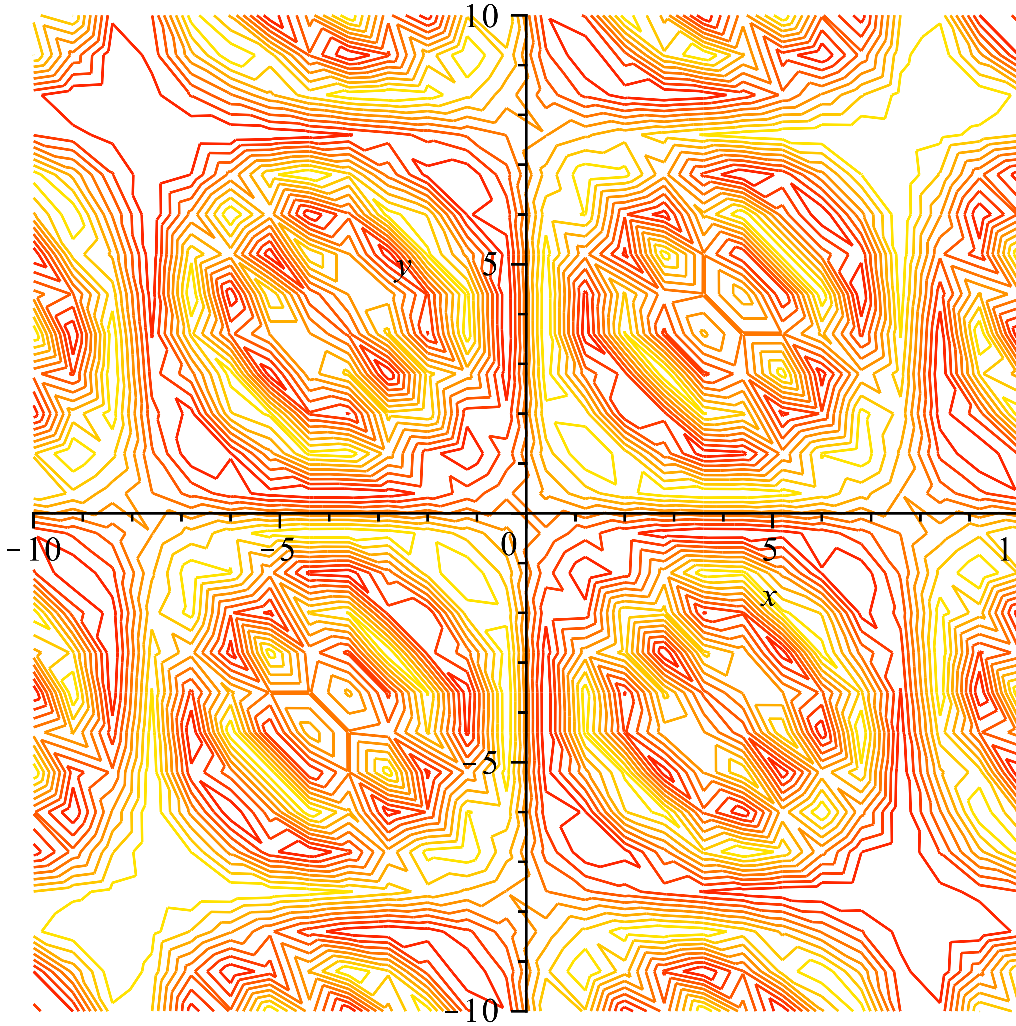
```
> plot3d({h(x),j(x)},x=-3..3,y=-3..3,
  title=`Interseccion de h(x) y j(x)`);
```

Interseccion de h(x) y j(x)



Mapas Topológicos

```
> with(plots):
> contourplot(sin(x*y),x=-10..10,y=-10.
  .10);
```



Graficando Campos Vectoriales :

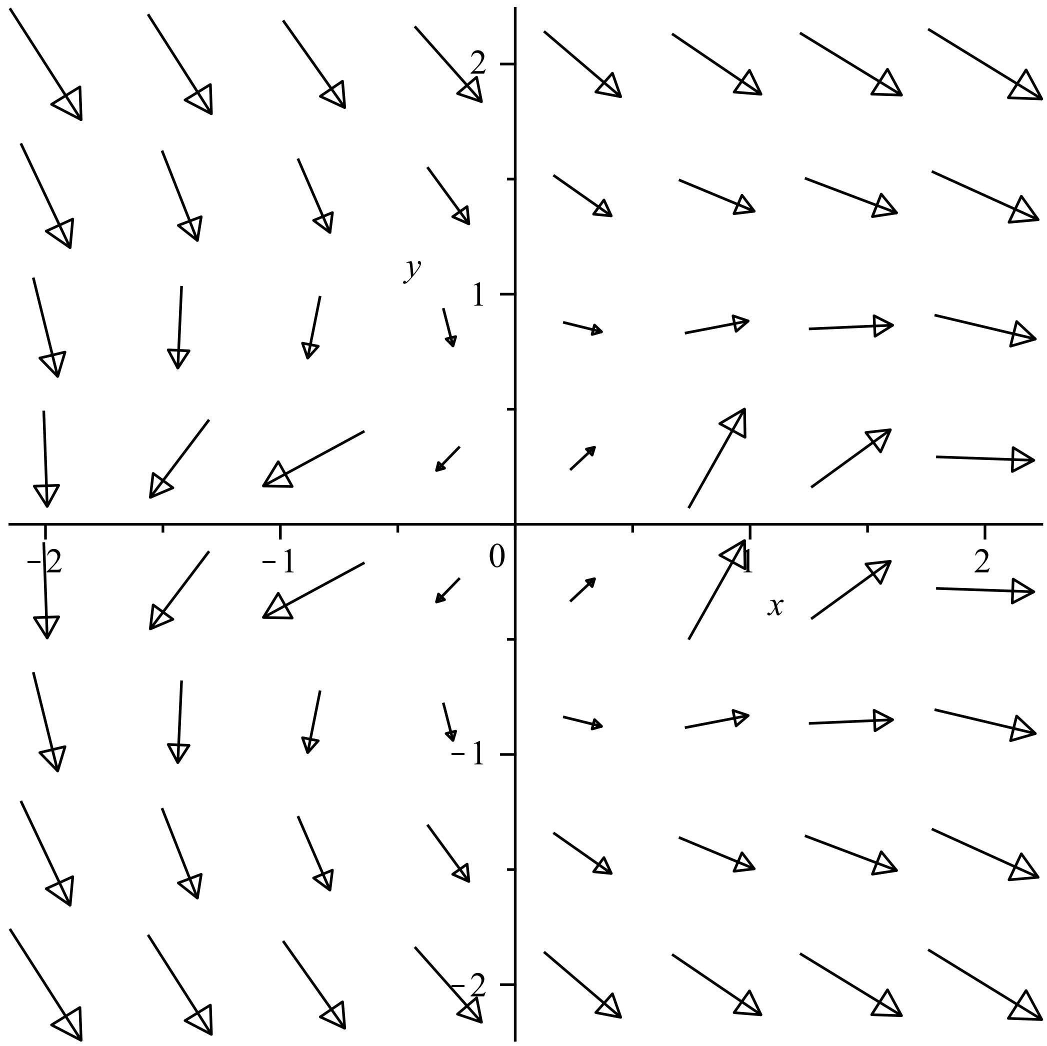
```
> phi[1]:=(x,y)->ln((x^2+2*x+1+y^2)^(1/2));
```

$$\phi_1 := (x, y) \rightarrow \ln\left(\sqrt{x^2 + 2x + 1 + y^2}\right) \tag{10.6}$$

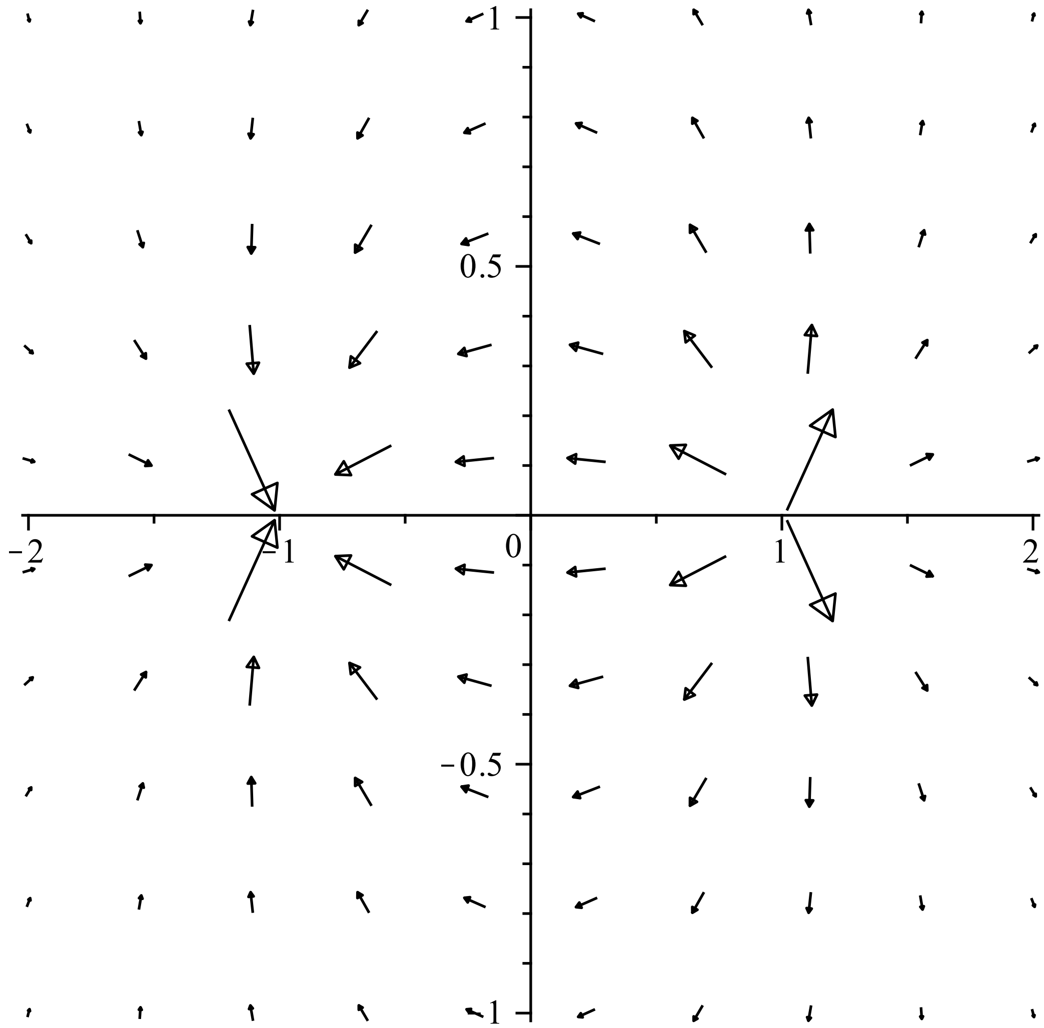
```
> phi[2]:=(x,y)->-ln((x^2-2*x+1+y^2)^(1/2));
```

$$\phi_2 := (x, y) \rightarrow -\ln\left(\sqrt{x^2 - 2x + 1 + y^2}\right) \tag{10.7}$$

```
> fieldplot([phi[1](x,y),phi[2](x,y)],x=-2..2,y=-2..2,arrows=SLIM,grid=[8,8]);
```

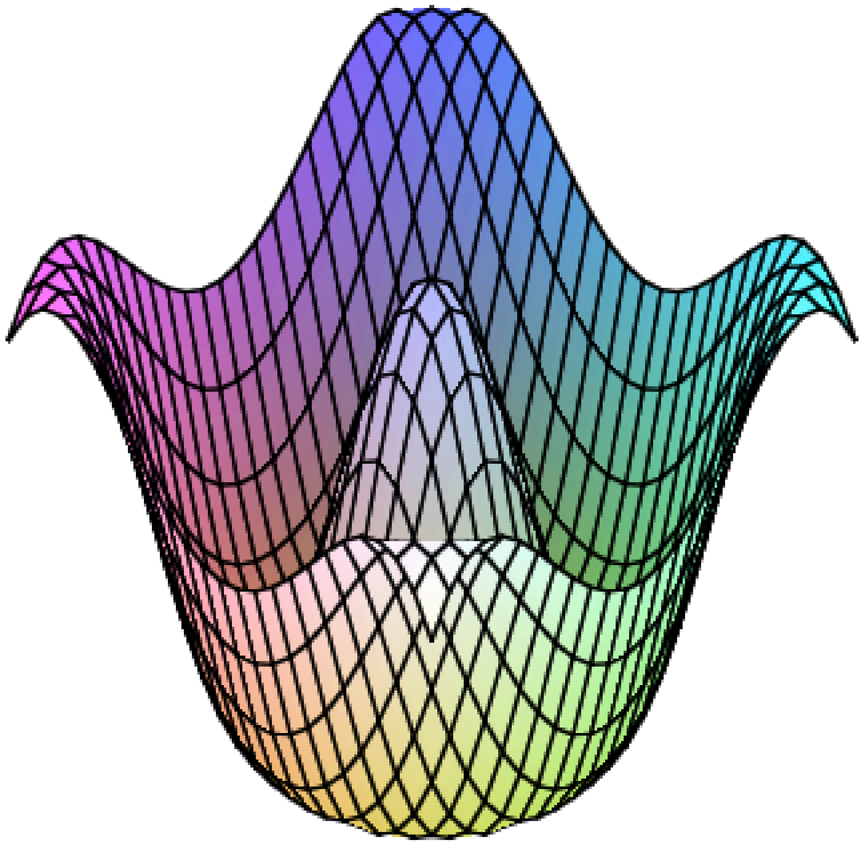



```
> gradplot((-phi[1](x,y)-phi[2](x,y)),x=
-2..2,y=-1..1,arrows=SLIM,grid=[10,10]
);
```



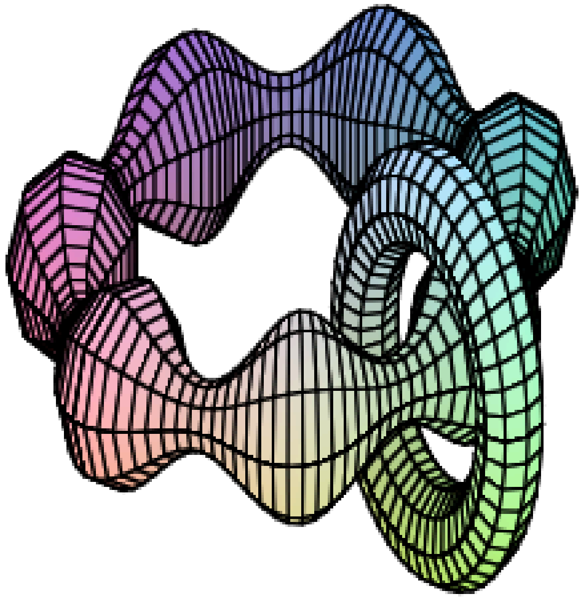
Animaciones:

```
> animate3d(cos(sqrt(t*x^2+t*y^2)),x=-5.
  .5,y=-5..5,t=1..8);
```



Con un poco más de complejidad

```
> tubeplot({[10*cos(t),10*sin(t),0,t=0.
.2*Pi,radius=2+cos(7*t),numpoints=120,
tubepoint=24],[0,10+5*cos(t),5*sin(t),
t=0..2*Pi,radius=1.5,numpoint=50,
tubepoint=18]});
```



9 Aplicaciones

Algebra Lineal

```
> restart:
> with(linalg):# Leemos el paquete de A.L.
```

Creando Matrices y Vectores

```
> A:=array( [ [a, b, c], [d, e, f ], [g, h, i ] ] );
```

$$A := \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \textbf{(11.1.1.1)}$$

```
> B:=array(antisymmetric,1..3,1..3):
> B[1,2]:=x+a; B[1,3]:=y+b; B[2,3]:=
  z+c;
```

$$B_{1,2} := x + a$$

$$B_{1,3} := y + b$$

$$B_{2,3} := z + c \qquad (11.1.1.2)$$

```
> print(B);
```

$$\begin{bmatrix} 0 & x+a & y+b \\ -x-a & 0 & z+c \\ -y-b & -z-c & 0 \end{bmatrix} \qquad (11.1.1.3)$$

```
> v:= vector([1,2,3] );
```

$$v := \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \qquad (11.1.1.4)$$

```
> C:=matrix(4,4, (i,j) -> i^(j-1));
```

$$C := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix} \qquad (11.1.1.5)$$

```
> M:=array(antisymmetric,1..3,1..3):
Warning,  computation interrupted
enter element 1,2 > 2*alpha
Warning, inserted missing semicolon at end of statement
```

$$2\,\alpha \qquad (11.1.1.6)$$

```
enter element 1,3 > 2*beta;
```

$$2\,\beta \qquad (11.1.1.7)$$

```
enter element 2,3 > 3*gamma;
```

$$3\,\gamma \qquad (11.1.1.8)$$

```
> N:=diag(mu,nu,lambda);
```

$$N := \begin{bmatrix} \mu & 0 & 0 \\ 0 & \nu & 0 \\ 0 & 0 & \lambda \end{bmatrix} \qquad (11.1.1.9)$$

```
> G:=diag(M,N);
```

$$G := \begin{bmatrix} 0 & M_{1,2} & M_{1,3} & 0 & 0 & 0 \\ -M_{1,2} & 0 & M_{2,3} & 0 & 0 & 0 \\ -M_{1,3} & -M_{2,3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \nu & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix} \quad (11.1.1.10)$$

La función "solve" para resolver un sistema de ecuaciones:

```
> ec1:= (c^2-1)*x+(c-1)*y = (1-c)^2 ;
```

$$ec1 := (c^2-1)\,x + (c-1)\,y = (1-c)^2 \quad (11.1.2.1)$$

```
> ec2:=(c-1)*x+(c^2-1)*y=c-1;
```

$$ec2 := (c-1)\,x + (c^2-1)\,y = c-1 \quad (11.1.2.2)$$

```
> sis:=({ec1,ec2}):
> solve(sis,{x,y});
```

$$\left\{x = \frac{c^2-2}{c\,(c+2)},\, y = \frac{2}{c\,(c+2)}\right\} \quad (11.1.2.3)$$

Algebra de matrices para resolver el sistema de ecuaciones

```
> H:=genmatrix(sis,[x,y],'flag');
```

$$H := \begin{bmatrix} c-1 & c^2-1 & c-1 \\ c^2-1 & c-1 & (1-c)^2 \end{bmatrix} \quad (11.1.3.1)$$

```
> A:=genmatrix(sis,[x,y]);
```

$$A := \begin{bmatrix} c-1 & c^2-1 \\ c^2-1 & c-1 \end{bmatrix} \quad (11.1.3.2)$$

```
> B:=delcols(H,1..2);
```

$$B := \begin{bmatrix} c-1 \\ (1-c)^2 \end{bmatrix} \quad (11.1.3.3)$$

La ecuacion A X = B se puede resolver con:

```
> sol:= linsolve(A,B);
```

(11.1.3.4)

$$sol := \begin{bmatrix} \frac{c^2-2}{c(c+2)} \\ \frac{2}{c(c+2)} \end{bmatrix} \quad (11.1.3.4)$$

```
> x:=sol[1,1]; y:=sol[2,1];
```

$$x := \frac{c^2-2}{c(c+2)}$$

$$y := \frac{2}{c(c+2)} \quad (11.1.3.5)$$

```
> simplify(ec1); simplify(ec2);
```

$$1-2c+c^2 = (c-1)^2$$

$$c-1 = c-1 \quad (11.1.3.6)$$

```
> x:='x': y:='y': # Limpio las variables x y
```

Operaciones con matrices

```
> S:=evalm(M+N);
```

$$S := \begin{bmatrix} \mu & M_{1,2} & M_{1,3} \\ -M_{1,2} & \nu & M_{2,3} \\ -M_{1,3} & -M_{2,3} & \lambda \end{bmatrix} \quad (11.1.4.1)$$

```
> evalm(3*M + 5/3*N);
```

$$\begin{bmatrix} \frac{5}{3}\mu & 3M_{1,2} & 3M_{1,3} \\ -3M_{1,2} & \frac{5}{3}\nu & 3M_{2,3} \\ -3M_{1,3} & -3M_{2,3} & \frac{5}{3}\lambda \end{bmatrix} \quad (11.1.4.2)$$

```
> evalm(M*N);
Error, (in evalm/evaluate) use the &* operator for matrix/vector multiplication
```

Forma incorrecta de multiplicar matrices

```
> evalm(M &* N);
```

(11.1.4.3)

$$\begin{bmatrix} 0 & M_{1,2}\nu & M_{1,3}\lambda \\ -M_{1,2}\mu & 0 & M_{2,3}\lambda \\ -M_{1,3}\mu & -M_{2,3}\nu & 0 \end{bmatrix} \quad \textbf{(11.1.4.3)}$$

```
> evalm(N &* M);
```

$$\begin{bmatrix} 0 & M_{1,2}\mu & M_{1,3}\mu \\ -M_{1,2}\nu & 0 & M_{2,3}\nu \\ -M_{1,3}\lambda & -M_{2,3}\lambda & 0 \end{bmatrix} \quad \textbf{(11.1.4.4)}$$

```
> V:= vector([x,y,z]);
```

$$V := \begin{bmatrix} x & y & z \end{bmatrix} \quad \textbf{(11.1.4.5)}$$

```
> evalm(M &* V);
```

$$\begin{bmatrix} M_{1,2}\,y + M_{1,3}\,z & -M_{1,2}\,x + M_{2,3}\,z & -M_{1,3}\,x - M_{2,3}\,y \end{bmatrix} \quad \textbf{(11.1.4.6)}$$

```
> S1:= evalm( (M)^(3) );
```

$$S1 := \Big[\Big[0, \left(-M_{1,2}^2 - M_{1,3}^2\right) M_{1,2} - M_{1,2}\,M_{2,3}^2, \left(-M_{1,2}^2 - M_{1,3}^2\right) M_{1,3} - M_{1,3}\,M_{2,3}^2\Big], \Big[-\left(-M_{1,2}^2 - M_{2,3}^2\right) M_{1,2} + M_{1,2}\,M_{1,3}^2, 0, -M_{1,3}^2\,M_{2,3} + \left(-M_{1,2}^2 - M_{2,3}^2\right) M_{2,3}\Big], \Big[M_{1,2}^2\,M_{1,3} - \left(-M_{1,3}^2 - M_{2,3}^2\right) M_{1,3}, M_{1,2}^2\,M_{2,3} - \left(-M_{1,3}^2 - M_{2,3}^2\right) M_{2,3}, 0\Big]\Big] \quad \textbf{(11.1.4.7)}$$

```
> S1:= map(factor,S1);
```

$$S1 := \Big[\Big[0, -M_{1,2}\left(M_{1,2}^2 + M_{1,3}^2 + M_{2,3}^2\right), -M_{1,3}\left(M_{1,2}^2 + M_{1,3}^2 + M_{2,3}^2\right)\Big], \Big[M_{1,2}\left(M_{1,2}^2 + M_{1,3}^2 + M_{2,3}^2\right), 0, -M_{2,3}\left(M_{1,2}^2 + M_{1,3}^2 + M_{2,3}^2\right)\Big], \Big[M_{1,3}\left(M_{1,2}^2 + M_{1,3}^2 + M_{2,3}^2\right), M_{2,3}\left(M_{1,2}^2 + M_{1,3}^2 + M_{2,3}^2\right), 0\Big]\Big] \quad \textbf{(11.1.4.8)}$$

```
> print(S);
```

$$\begin{bmatrix} \mu & M_{1,2} & M_{1,3} \\ -M_{1,2} & \nu & M_{2,3} \\ -M_{1,3} & -M_{2,3} & \lambda \end{bmatrix} \quad \textbf{(11.1.4.9)}$$

```
> zeta:=inverse(S);
```

$$\zeta := \left[\left[\frac{\nu\lambda + M_{2,3}^2}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}, -\frac{M_{1,2}\lambda + M_{1,3}M_{2,3}}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}, -\frac{-M_{1,2}M_{2,3} + M_{1,3}\nu}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}\right], \left[\frac{M_{1,2}\lambda - M_{1,3}M_{2,3}}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}, \frac{\mu\lambda + M_{1,3}^2}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}, -\frac{\mu M_{2,3} + M_{1,2}M_{1,3}}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}\right], \left[\frac{M_{1,2}M_{2,3} + M_{1,3}\nu}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}, \frac{\mu M_{2,3} - M_{1,2}M_{1,3}}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}, \frac{\mu\nu + M_{1,2}^2}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu}\right]\right] \tag{11.1.4.10}$$

```
> map(simplify,evalm(S &*  zeta));
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11.1.4.11}$$

```
> trace(zeta);
```

$$\frac{\nu\lambda + M_{2,3}^2}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu} + \frac{\mu\lambda + M_{1,3}^2}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu} + \frac{\mu\nu + M_{1,2}^2}{\mu\nu\lambda + \mu M_{2,3}^2 + M_{1,2}^2\lambda + M_{1,3}^2\nu} \tag{11.1.4.12}$$

```
> det(zeta);
```

$$\frac{1}{\mu\,\nu\,\lambda+\mu\,M_{2,3}^2+M_{1,2}^2\,\lambda+M_{1,3}^2\,\nu} \tag{11.1.4.13}$$

```
> normal( trace(zeta) / det(zeta) );
```

$$\nu\,\lambda+M_{2,3}^2+\mu\,\lambda+M_{1,3}^2+\mu\,\nu+M_{1,2}^2 \tag{11.1.4.14}$$

Cálculo de Autovectores y Autovalores

```
> K:=matrix([[sqrt(2),alpha,beta],
[alpha,sqrt(2),alpha],[beta,alpha,
sqrt(2)]]);
```

$$K := \begin{bmatrix} \sqrt{2} & \alpha & \beta \\ \alpha & \sqrt{2} & \alpha \\ \beta & \alpha & \sqrt{2} \end{bmatrix} \tag{11.1.5.1}$$

```
> pc:=charpoly(K, lambda); #
Polinomio Caracteristico
```

$$pc := \lambda^3-3\,\lambda^2\sqrt{2}+6\,\lambda-2\,\lambda\,\alpha^2-2\sqrt{2}+2\sqrt{2}\,\alpha^2-2\,\beta\,\alpha^2-\beta^2\,\lambda+\beta^2\sqrt{2} \tag{11.1.5.2}$$

```
> factor(%);
```

$$-\left(2-2\,\lambda\sqrt{2}+\beta\sqrt{2}+\lambda^2-\beta\,\lambda-2\,\alpha^2\right)\left(\sqrt{2}-\beta-\lambda\right) \tag{11.1.5.3}$$

```
> solve(pc=0,lambda);
```

$$\sqrt{2}-\beta,\ \frac{1}{2}\,\beta+\sqrt{2}+\frac{1}{2}\sqrt{\beta^2+8\,\alpha^2},\ \frac{1}{2}\,\beta+\sqrt{2}-\frac{1}{2}\sqrt{\beta^2+8\,\alpha^2} \tag{11.1.5.4}$$

```
> eigenvals(K);
```

$$\sqrt{2}-\beta,\ \frac{1}{2}\,\beta+\sqrt{2}+\frac{1}{2}\sqrt{\beta^2+8\,\alpha^2},\ \frac{1}{2}\,\beta+\sqrt{2}-\frac{1}{2}\sqrt{\beta^2+8\,\alpha^2} \tag{11.1.5.5}$$

```
> eigenvects(K);
```

$$\left[\frac{1}{2}\,\beta+\sqrt{2}+\frac{1}{2}\sqrt{\beta^2+8\,\alpha^2},\ 1,\ \left\{\left[\begin{matrix} 1 & -\dfrac{\frac{1}{2}\,\beta-\frac{1}{2}\sqrt{\beta^2+8\,\alpha^2}}{\alpha} & 1 \end{matrix}\right]\right\}\right], \tag{11.1.5.6}$$

$$\left[\frac{1}{2}\,\beta+\sqrt{2}-\frac{1}{2}\sqrt{\beta^2+8\,\alpha^2},\ 1,\right.$$

$$\left\{\left[\begin{array}{ccc} 1 & -\dfrac{\frac{1}{2}\beta+\frac{1}{2}\sqrt{\beta^2+8\alpha^2}}{\alpha} & 1 \end{array}\right]\right\}\right], \left[\sqrt{2}-\beta, 1, \left\{\left[\begin{array}{ccc} -1 & 0 & 1 \end{array}\right]\right\}\right]$$

El resultado es una lista de la forma:

[e_i, m_i, { v[1,i],... v[n_i,i] }]

donde:

e_i = son los autovalores

m_i = multiplicidad

{v[1,i], ..., v[n_i,i]} = conjunto de vectores bases

```
> print(K);
```

$$\left[\begin{array}{ccc} \sqrt{2} & \alpha & \beta \\ \alpha & \sqrt{2} & \alpha \\ \beta & \alpha & \sqrt{2} \end{array}\right] \quad \textbf{(11.1.5.7)}$$

Solución de B x = v mediante Gauss-Jordan

```
> B:=matrix(3,3,[19,-50,88,  53,85,
  -49, 78, 17, 72] );
```

$$B := \left[\begin{array}{ccc} 19 & -50 & 88 \\ 53 & 85 & -49 \\ 78 & 17 & 72 \end{array}\right] \quad \textbf{(11.1.6.1)}$$

```
> v1:= vector( [3 ,5,-2] ):
> v2:= vector( [4 ,-5,9] ):
> v3:= vector( [21 ,4,7] ):
> augment(B,v1,v2,v3 );
```

$$\left[\begin{array}{cccccc} 19 & -50 & 88 & 3 & 4 & 21 \\ 53 & 85 & -49 & 5 & -5 & 4 \\ 78 & 17 & 72 & -2 & 9 & 7 \end{array}\right] \quad \textbf{(11.1.6.2)}$$

Construimos la matriz aumentada, e invocamos la solución

```
> gaussjord(%);
```

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & \frac{56399}{9855} & -\frac{42938}{9855} & \frac{43729}{3285} \\ 0 & 1 & 0 & -\frac{20528}{3285} & \frac{15761}{3285} & -\frac{15913}{1095} \\ 0 & 0 & 1 & -\frac{46832}{9855} & \frac{36584}{9855} & -\frac{35782}{3285} \end{array}\right] \quad (11.1.6.3)$$

```
> leastsqrs(B, v1); leastsqrs(B, v2);
  leastsqrs(B, v3);
```

$$\left[\begin{array}{ccc} \frac{56399}{9855} & -\frac{20528}{3285} & -\frac{46832}{9855} \end{array}\right]$$

$$\left[\begin{array}{ccc} -\frac{42938}{9855} & \frac{15761}{3285} & \frac{36584}{9855} \end{array}\right]$$

$$\left[\begin{array}{ccc} \frac{43729}{3285} & -\frac{15913}{1095} & -\frac{35782}{3285} \end{array}\right] \quad (11.1.6.4)$$

Funciones test

```
> beta := sqrt(2)*sqrt(5-sqrt(5));
```

$$\beta := \sqrt{2}\,\sqrt{5-\sqrt{5}} \quad (11.1.7.1)$$

```
> A:= matrix([[(sqrt(5)+beta +1)/4, -
  beta/2 ],[ beta/4 ,( sqrt(5) -
  beta+1)/4]]);
```

$$A := \left[\begin{array}{cc} \frac{1}{4}\sqrt{5} + \frac{1}{4}\sqrt{2}\,\sqrt{5-\sqrt{5}} + \frac{1}{4} & -\frac{1}{2}\sqrt{2}\,\sqrt{5-\sqrt{5}} \\ \frac{1}{4}\sqrt{2}\,\sqrt{5-\sqrt{5}} & \frac{1}{4}\sqrt{5} - \frac{1}{4}\sqrt{2}\,\sqrt{5-\sqrt{5}} + \frac{1}{4} \end{array}\right] \quad (11.1.7.2)$$

```
> B:=matrix( [[(sqrt(5)+1)/4,-beta/4
  ],[ beta/4,(sqrt(5)+1)/4 ]  ]);
```

$$B := \left[\begin{array}{cc} \frac{1}{4}\sqrt{5} + \frac{1}{4} & -\frac{1}{4}\sqrt{2}\,\sqrt{5-\sqrt{5}} \\ \frac{1}{4}\sqrt{2}\,\sqrt{5-\sqrt{5}} & \frac{1}{4}\sqrt{5} + \frac{1}{4} \end{array}\right] \quad (11.1.7.3)$$

Queremos verificar si A y B son similares, es decir, si existe una matriz T, no singular, de manera que

B = T A T^(-1).

```
> issimilar(A,B,'T');
```

$$true \quad (11.1.7.4)$$

```
> map(simplify,T);
```

$$\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \quad (11.1.7.5)$$

Cálculo Vectorial.

Gradiente

```
> f:= 4*x*y*z - 5*y*x^3;
```

$$f := 4\,x\,y\,z - 5\,y\,x^3 \quad (11.1.8.1)$$

```
> gradf:= grad(f,[x,y,z]);
```

$$gradf := \begin{bmatrix} 4\,y\,z - 15\,y\,x^2 & 4\,x\,z - 5\,x^3 & 4\,x\,y \end{bmatrix} \quad (11.1.8.2)$$

Vectores

```
> v:=vector([4*x-3*x^3*y,7*x*y*z^2+5*
  y^3,4*x^2*y^2+2*x]);
```

$$v := \begin{bmatrix} 4\,x - 3\,y\,x^3 & 7\,x\,y\,z^2 + 5\,y^3 & 4\,x^2\,y^2 + 2\,x \end{bmatrix} \quad (11.1.8.3)$$

Rotores

```
> curlv:= curl(v,[x,y,z] );
```

$$curlv := \begin{bmatrix} 8\,y\,x^2 - 14\,x\,y\,z & -2 - 8\,x\,y^2 & 7\,y\,z^2 + 3\,x^3 \end{bmatrix} \quad (11.1.8.4)$$

Jacobianos

```
> jacobian(%,[x,y,z]);
```

$$\begin{bmatrix} 16\,x\,y - 14\,y\,z & 8\,x^2 - 14\,x\,z & -14\,x\,y \\ -8\,y^2 & -16\,x\,y & 0 \\ 9\,x^2 & 7\,z^2 & 14\,y\,z \end{bmatrix} \quad (11.1.8.5)$$

Laplacianos

```
> laplacf:= laplacian(f,[x,y,z] );
```

$$laplacf := -30\,x\,y \quad (11.1.8.6)$$

Evaluación boleana de expresiones

```
> evalb(laplacf = diverge(gradf,[x,y,
```

```
z]));
```

$$true \tag{11.1.8.7}$$

Diferentes tipos de coordenadas

```
> f := r*sin(theta)*z^2:  v := [r,
theta, z]:
> gr:=grad(f, v, coords=cylindrical);
```

$$gr := \left[\begin{array}{ccc} \sin(\theta)\, z^2 & \cos(\theta)\, z^2 & 2\, r \sin(\theta)\, z \end{array}\right] \tag{11.1.8.8}$$

```
> diverge(gr,v, coords=spherical);
```

$$\frac{\sin(\theta)^2 z^2 + \cos(\theta)^2 z^2 + 2\, r \sin(\theta)}{r \sin(\theta)} \tag{11.1.8.9}$$

```
> laph= laplacian(f,v);
```

$$laph = -r \sin(\theta)\, z^2 + 2\, r \sin(\theta) \tag{11.1.8.10}$$

```
> laph= laplacian(f,v,coords=
bipolarcylindrical);
```

$$laph = \left(\cosh(\theta) - \cos(r)\right)^2 \left(-r \sin(\theta)\, z^2 + \frac{2\, r \sin(\theta)}{\left(\cosh(\theta) - \cos(r)\right)^2}\right) \tag{11.1.8.11}$$

```
> ?coords
```

==