

```
> restart: with(plots);
```

```
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d,  
conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot,  
display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d,  
inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d,  
listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto,  
plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d,  
polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions,  
setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]
```

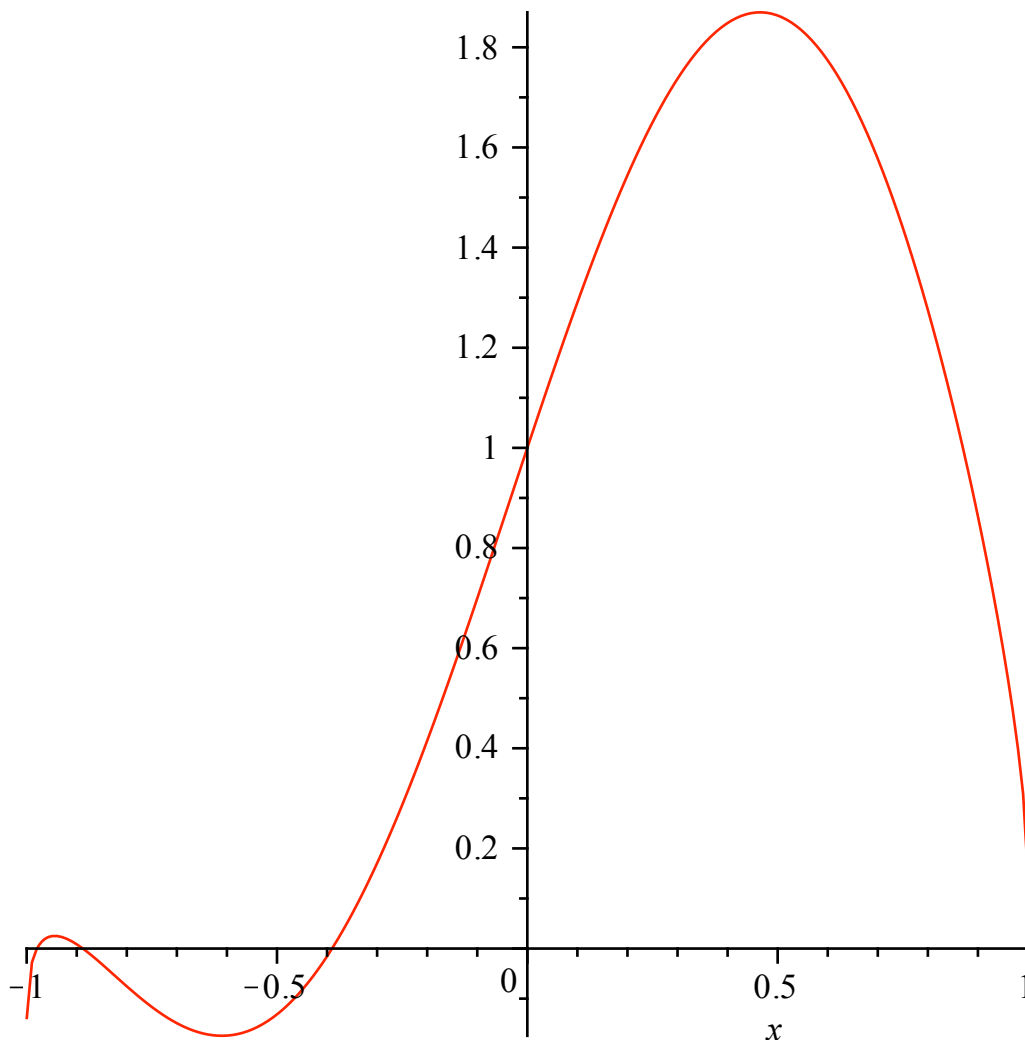
(1)

```
> f := x -> sqrt(1-x^2)+sin(3*x);
```

$$f := x \rightarrow \sqrt{1 - x^2} + \sin(3x)$$

(2)

```
> plot(f(x), x=-1..1);
```



```
> n:=7;
```

```
n := 7
```

(3)

```
> AproxPot[n] := taylor(f(x), x=0, n);
```

$$\text{AproxPot}_7 := 1 + 3x - \frac{1}{2}x^2 - \frac{9}{2}x^3 - \frac{1}{8}x^4 + \frac{81}{40}x^5 - \frac{1}{16}x^6 + O(x^7)$$

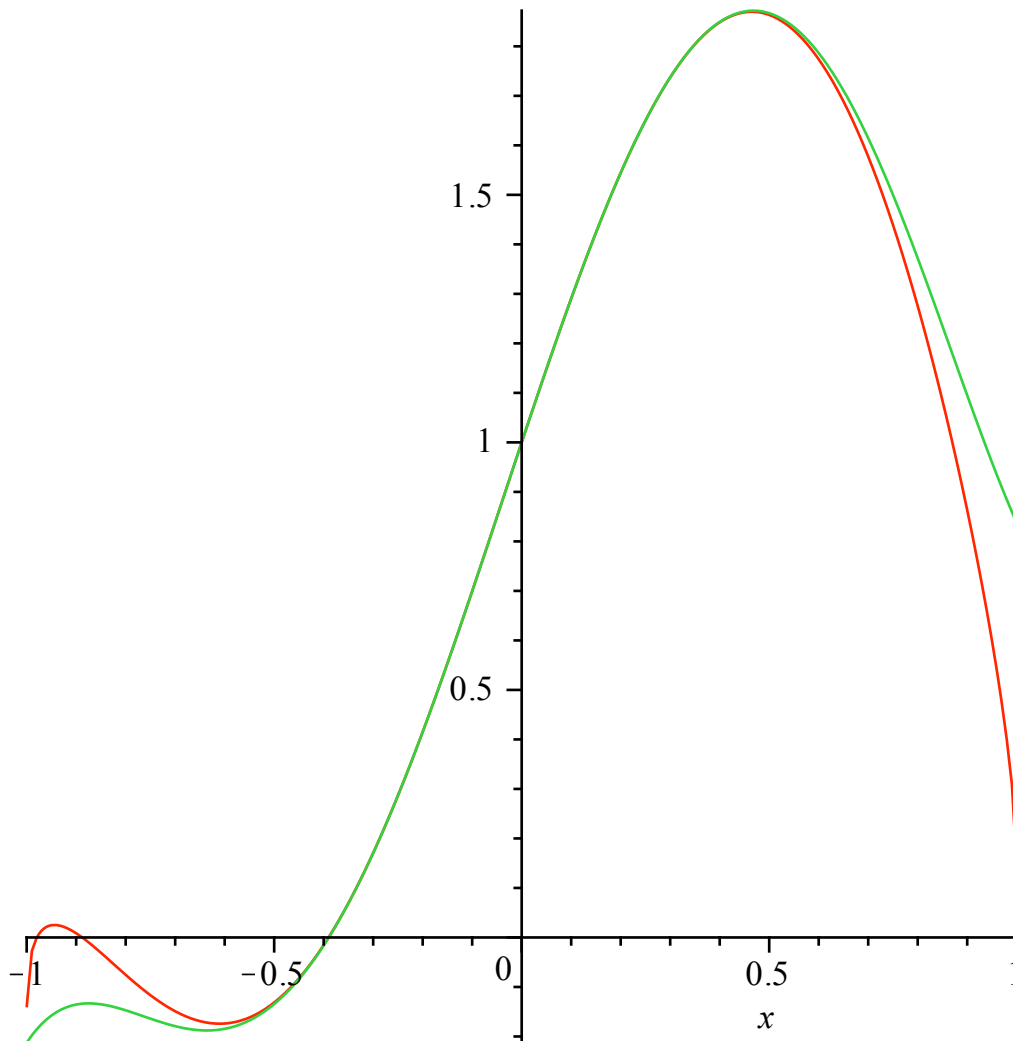
(4)

```
> PlotFunc[n] := convert(AproxPot[n], polynom);
```

$$\text{PlotFunc}_7 := 1 + 3x - \frac{1}{2}x^2 - \frac{9}{2}x^3 - \frac{1}{8}x^4 + \frac{81}{40}x^5 - \frac{1}{16}x^6$$

(5)

```
> plot([f(x), PlotFunc[n]], x=-1..1);
```



```
> Int(x^l*x^m, x=-1..1) = int(x^l*x^m, x=-1..1);
```

$$\int_{-1}^1 x^l x^m dx = \frac{1 + (-1)^{l+m}}{l+m+1}$$

(6)

```
> restart: with(plots): with(orthopoly);
```

```
[G, H, L, P, T, U]
```

(7)

```
> P(0, x); P(1, x); P(2, x); P(3, x); P(4, x); P(5, x);
```

```
1  
x
```

$$\begin{aligned}
 & -\frac{1}{2} + \frac{3}{2}x^2 \\
 & \frac{5}{2}x^3 - \frac{3}{2}x \\
 & \frac{3}{8} + \frac{35}{8}x^4 - \frac{15}{4}x^2 \\
 & \frac{63}{8}x^5 - \frac{35}{4}x^3 + \frac{15}{8}x
 \end{aligned} \tag{8}$$

> `int(P(n,x)*P(m,x),x=-1..1);`

$$\int_{-1}^1 P(n,x) P(m,x) dx \tag{9}$$

> `int(P(3,x)*P(4,x),x=-1..1);`

$$0 \tag{10}$$

> `int(P(3,x)*P(3,x),x=-1..1);`

$$\frac{2}{7} \tag{11}$$

> `f := x -> sqrt(1-x^2)+sin(3*x);`

$$f := x \rightarrow \sqrt{1-x^2} + \sin(3x) \tag{12}$$

> `Int(f(x)*P(0,x),x=-1..1)/Int(P(0,x)*P(0,x),x=-1..1)=int(f(x)*P(0,x),x=-1..1)/int(P(0,x)*P(0,x),x=-1..1);`

$$\frac{\int_{-1}^1 (\sqrt{1-x^2} + \sin(3x)) dx}{\int_{-1}^1 1 dx} = \frac{1}{4} \pi \tag{13}$$

> `Int(f(x)*P(1,x),x=-1..1)/Int(P(1,x)*P(1,x),x=-1..1)=int(f(x)*P(1,x),x=-1..1)/int(P(1,x)*P(1,x),x=-1..1);`

$$\frac{\int_{-1}^1 (\sqrt{1-x^2} + \sin(3x)) x dx}{\int_{-1}^1 x^2 dx} = -\cos(3) + \frac{1}{3} \sin(3) \tag{14}$$

> `Int(f(x)*P(2,x),x=-1..1)/Int(P(2,x)*P(2,x),x=-1..1)=int(f(x)*P(2,x),x=-1..1)/int(P(2,x)*P(2,x),x=-1..1);`

$$\frac{\int_{-1}^1 (\sqrt{1-x^2} + \sin(3x)) \left(-\frac{1}{2} + \frac{3}{2}x^2\right) dx}{\int_{-1}^1 \left(-\frac{1}{2} + \frac{3}{2}x^2\right)^2 dx} = -\frac{5}{32} \pi \tag{15}$$

> `Int(f(x)*P(3,x),x=-1..1)/Int(P(3,x)*P(3,x),x=-1..1)=int(f(x)*P(3,x),x=-1..1)/int(P(3,x)*P(3,x),x=-1..1);`

$$\frac{\int_{-1}^1 (\sqrt{1-x^2} + \sin(3x)) \left(\frac{5}{2} x^3 - \frac{3}{2} x \right) dx}{\int_{-1}^1 \left(\frac{5}{2} x^3 - \frac{3}{2} x \right)^2 dx} = \frac{14}{9} \cos(3) + \frac{91}{27} \sin(3) \quad (16)$$

> `Int(f(x)*P(4,x),x=-1..1)/Int(P(4,x)*P(4,x),x=-1..1)=int(f(x)*P(4,x),x=-1..1)/int(P(4,x)*P(4,x),x=-1..1);`

$$\frac{\int_{-1}^1 (\sqrt{1-x^2} + \sin(3x)) \left(\frac{3}{8} + \frac{35}{8} x^4 - \frac{15}{4} x^2 \right) dx}{\int_{-1}^1 \left(\frac{3}{8} + \frac{35}{8} x^4 - \frac{15}{4} x^2 \right)^2 dx} = -\frac{9}{256} \pi \quad (17)$$

> `CoefLeg[k]:=int(f(x)*P(k,x),x=-1..1)/int(P(k,x)*P(k,x),x=-1..1);`

$$\text{CoefLeg}_k := \frac{\int_{-1}^1 (\sqrt{1-x^2} + \sin(3x)) P(k,x) dx}{\int_{-1}^1 P(k,x)^2 dx} \quad (18)$$

> `n:=7;AproxLeg[n]:=sum(CoefLeg[k]*P(k,x),k=0..n);`
`n:=7`

$$\begin{aligned} \text{AproxLeg}_7 := & \frac{1}{4} \pi + \frac{3}{2} \left(-\frac{2}{3} \cos(3) + \frac{2}{9} \sin(3) \right) x - \frac{5}{32} \pi \left(-\frac{1}{2} + \frac{3}{2} x^2 \right) \\ & + \frac{7}{2} \left(\frac{4}{9} \cos(3) + \frac{26}{27} \sin(3) \right) \left(\frac{5}{2} x^3 - \frac{3}{2} x \right) - \frac{9}{256} \pi \left(\frac{3}{8} + \frac{35}{8} x^4 - \frac{15}{4} x^2 \right) \\ & + \frac{11}{2} \left(-\frac{2}{3} \cos(3) - \frac{40}{9} \sin(3) \right) \left(\frac{63}{8} x^5 - \frac{35}{4} x^3 + \frac{15}{8} x \right) - \frac{65}{4096} \pi \left(-\frac{5}{16} \right. \\ & + \frac{231}{16} x^6 - \frac{315}{16} x^4 + \frac{105}{16} x^2 \left. \right) + \frac{15}{2} \left(\frac{674}{81} \cos(3) + \frac{14182}{243} \sin(3) \right) \left(\frac{429}{16} x^7 \right. \\ & \left. - \frac{693}{16} x^5 + \frac{315}{16} x^3 - \frac{35}{16} x \right) \end{aligned} \quad (19)$$

> `simplify(AproxLeg[n]);`

$$\begin{aligned} & \frac{20965}{65536} \pi - \frac{131131}{48} \cos(3) x^5 - \frac{2757755}{144} \sin(3) x^5 - \frac{15015}{65536} \pi x^6 + \frac{240955}{144} \cos(3) x^7 \\ & + \frac{5070065}{432} \sin(3) x^7 - \frac{63385}{432} x \cos(3) - \frac{1306445}{1296} x \sin(3) - \frac{13545}{65536} \pi x^2 \\ & + \frac{182105}{144} \cos(3) x^3 + \frac{3818815}{432} \sin(3) x^3 + \frac{10395}{65536} \pi x^4 \end{aligned} \quad (20)$$

> `plot([f(x),AproxLeg[n]],x=-1..1);`

